

# Linewidth-tolerant Feed-forward Dual-stage CPE Algorithm Based on 64-QAM Constellation Partitioning



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**Abstract** - A detailed analysis of a novel low-complexity two-stage digital feed-forward carrier recovery algorithm for 64-QAM, based on the rotation of constellation points to remove phase modulation is presented. Its performance and complexity is compared with previously proposed algorithms.

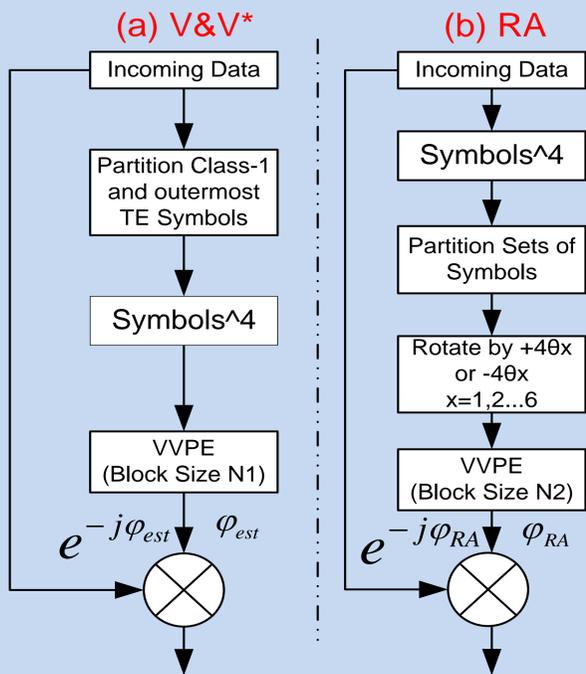


Figure 1: Block diagram of the proposed technique

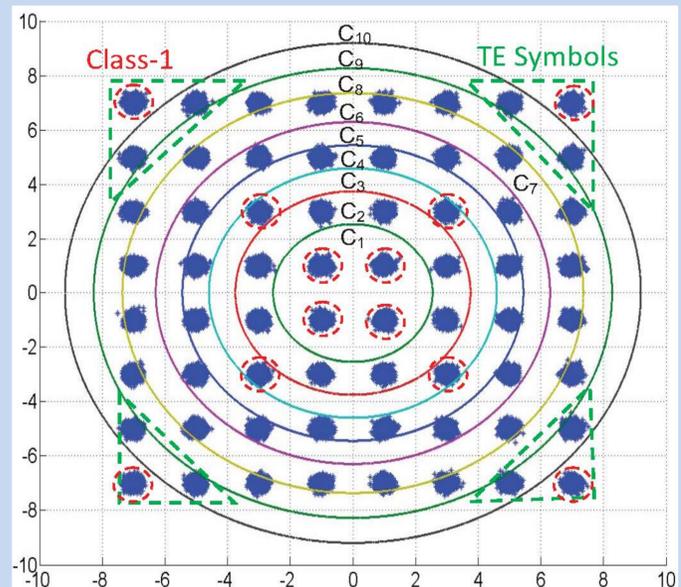


Figure 2: 64-QAM Constellation with different thresholds for separating symbols of different amplitudes. Symbols used in the first V&V\* stage are highlighted by red dashed circles (Class-1 Symbols) and green dashed triangles (Triangle Edge (TE) Symbols).

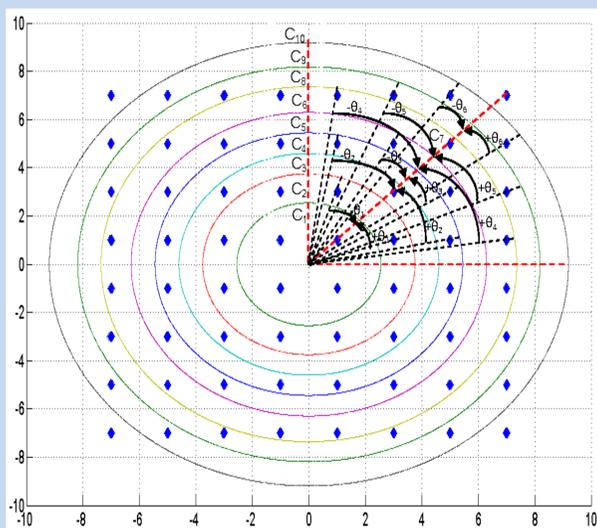


Figure 3: 64-QAM constellation showing all rings and rotation angles

- Symbols in the rings  $C_1$ ,  $C_3$ ,  $C_7$  and  $C_{10}$  are QPSK partitioned symbols that lie at modulation angles equal to  $\pi/4 + m \cdot \pi/2$  ( $m=0\dots3$ ).
- Symbols in the rings  $C_2$ ,  $C_4$ ,  $C_5$ ,  $C_6$ ,  $C_8$ , and  $C_9$  can be categorized into two sets of QPSK symbols with phase rotations  $\theta_x = \pm(\pi/4 - \tan^{-1}(k_x))$ , with  $k_x \in \{1/3, 1/5, 3/5, 1/7, 3/7, 5/7\}$ , with respect to the symbols lying in the rings  $C_1$ ,  $C_3$ ,  $C_7$  and  $C_{10}$ .

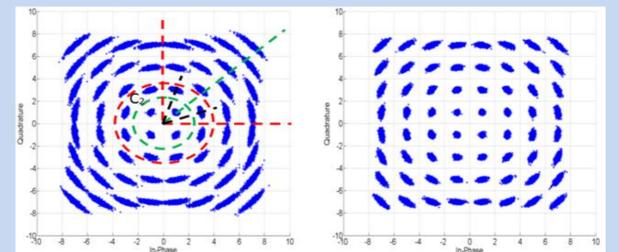


Figure 4: 64-QAM constellation after coarse (left) and fine (right) carrier phase estimation

- If the residual phase noise after the coarse carrier phase estimation is sufficiently small not to cross the boundaries between the symbols in corresponding rings, these symbols can be properly rotated by  $\theta_x$  in order to make them fall at an angle equal to  $\pi/4 + m \cdot \pi/2$  ( $m=0\dots3$ ).
- After the subsequent 4<sup>th</sup> power operation all the symbols will collapse down to unique positions and, having distinct thresholds, can be easily separated.
- Phase modulation of the rings  $C_1$ ,  $C_3$ ,  $C_7$  and  $C_{10}$  is directly removed while the phase modulation of the rings  $C_2$ ,  $C_4$ ,  $C_5$ ,  $C_6$ ,  $C_8$ , and  $C_9$  can be removed with:

$$RA_x = C_y \cdot \exp(4j\theta_x \cdot \text{sgn}(\text{Im}(C_y)))$$

where  $y = 2, 4, 5, 6, 8, 9$ ,  $x = 1, 2, \dots, 6$ ,  $\text{sgn}(\cdot)$  is the 'signum' function and  $\text{Im}(\cdot)$  is the imaginary part of the complex valued symbols

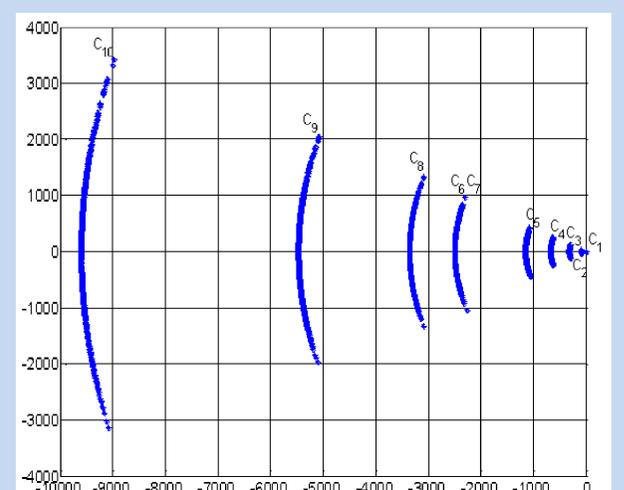


Figure 5: 64-QAM constellation after fourth power and rotation operation

## Simulation model

Received noisy samples:

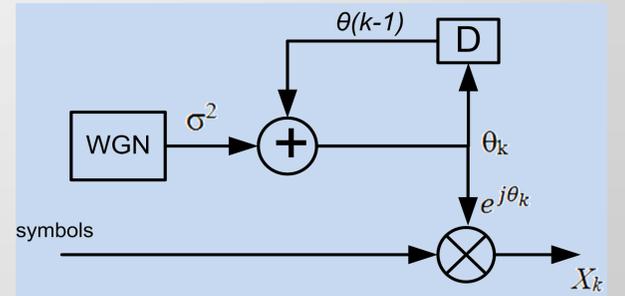
$$y_k = x_k e^{j\theta_k} + \eta_k$$

Phase noise:

$$\theta_k = \sum_{i=-\infty}^k v_i$$

$$\sigma_f^2 = 2\pi\Delta\nu T_s$$

- $x_k$  is the data symbol at time  $k$  that belongs to the set  $(\pm a \pm j \cdot b)$ , with  $a, b \in \{1, 3, 5, 7\}$ .
- $\eta_k$  is the AWG noise.
- $\theta_k$  is the laser phase noise, modeled as a Wiener process.
- $\Delta\nu$  is the combined laser linewidth of transmitter laser and LO.
- $T_s$  is the symbol period.



## Complexity analysis

CPE	Real Multipliers	Real Adders	Comparators	Look-Up Tables	Decisions
V&V*	8N	3N+2	4N+2	1	N
V&V*+CT	8N <sub>1</sub> +6N <sub>2</sub>	3N <sub>1</sub> +3N <sub>2</sub> +30	4N <sub>1</sub> +7	2	N <sub>2</sub>
V&V*+RA	8N <sub>1</sub> +6N <sub>2</sub> +36	3N <sub>1</sub> +3N <sub>2</sub> +4	4N <sub>1</sub> +13	3	N <sub>2</sub>
BPS	NM+2NM	2NM-M+3	M+1	0	NM+N
BPS+MLE	N <sub>1</sub> M+2N <sub>1</sub> M+N <sub>2</sub>	2N <sub>1</sub> M-M+N <sub>2</sub> +2	M+1	1	N <sub>1</sub> M+N <sub>2</sub>

Table 1: Computational complexity for various CPE algorithms

## Simulation Results

Case	2	4	6	7	8
CPE	V&V*	V&V*+CT	V&V*+RA	BPS	BPS+MLE
$\Delta\nu \cdot T_s$ @ 1dB penalty	$10^{-5}$	$3.7 \cdot 10^{-5}$	$3.7 \cdot 10^{-5}$	$5.7 \cdot 10^{-5}$	$5.4 \cdot 10^{-5}$
Equivalent linewidth @ 20 Gbaud	0.20 MHz	0.74 MHz	0.74 MHz	1.14 MHz	1.08 MHz

Table 2: Laser phase noise tolerances and their equivalent linewidths at 20 Gbaud

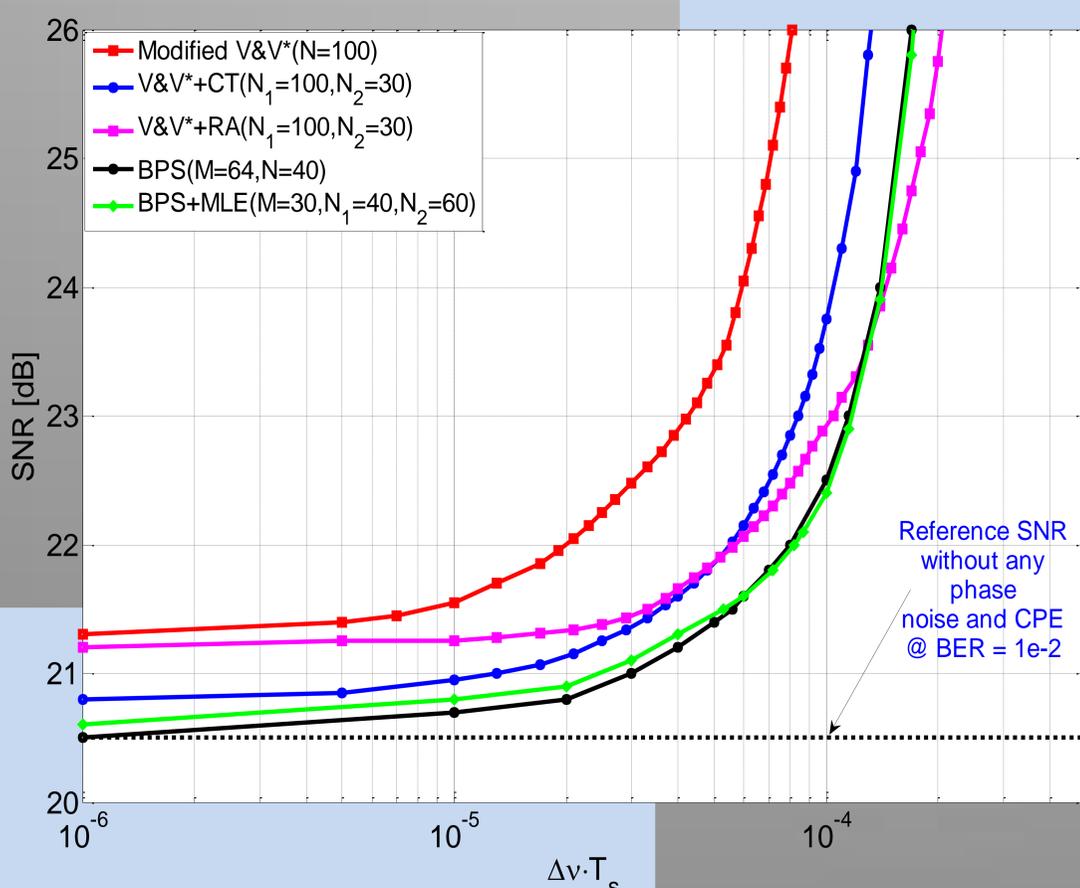


Figure 6: SNR vs. linewidth times symbol duration ( $\Delta\nu \cdot T_s$ ) product at  $BER=10^{-2}$  for different CPE schemes.

## Conclusions

A novel low complexity algorithm for carrier phase estimation of 64-QAM has been presented and its performance is analyzed through numerical simulations. At high phase noise values, performance of the proposed technique is even better than CT and BPS with approx. the same complexity as CT and almost 9 times less complexity than that of BPS.

