Channel Coding for Optical Communications

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speaker

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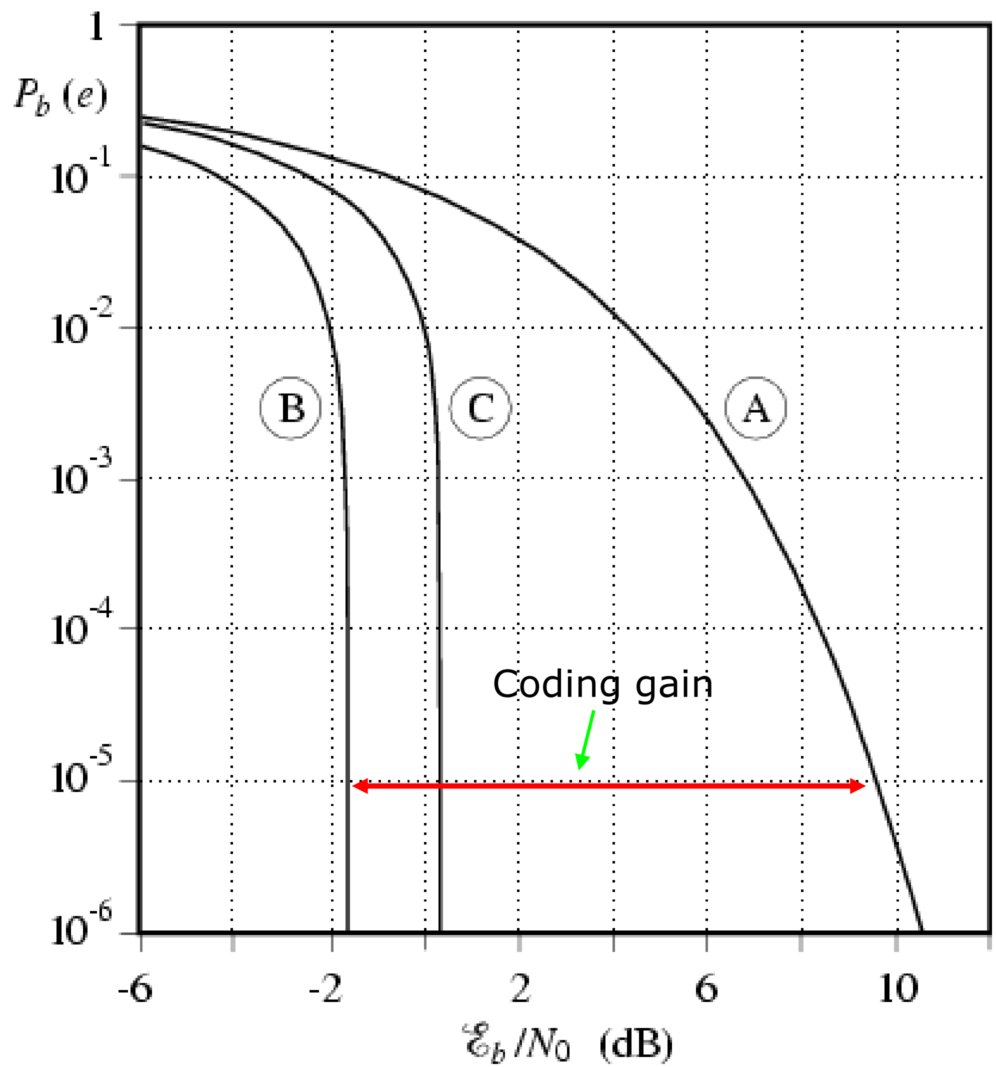
Summary

- Introduction
- The "standard" coding scheme and its avatars
- The impact of soft iterative decoding
- Turbo-product codes
- Low-density parity-check codes
- High-speed parallel decoder architectures

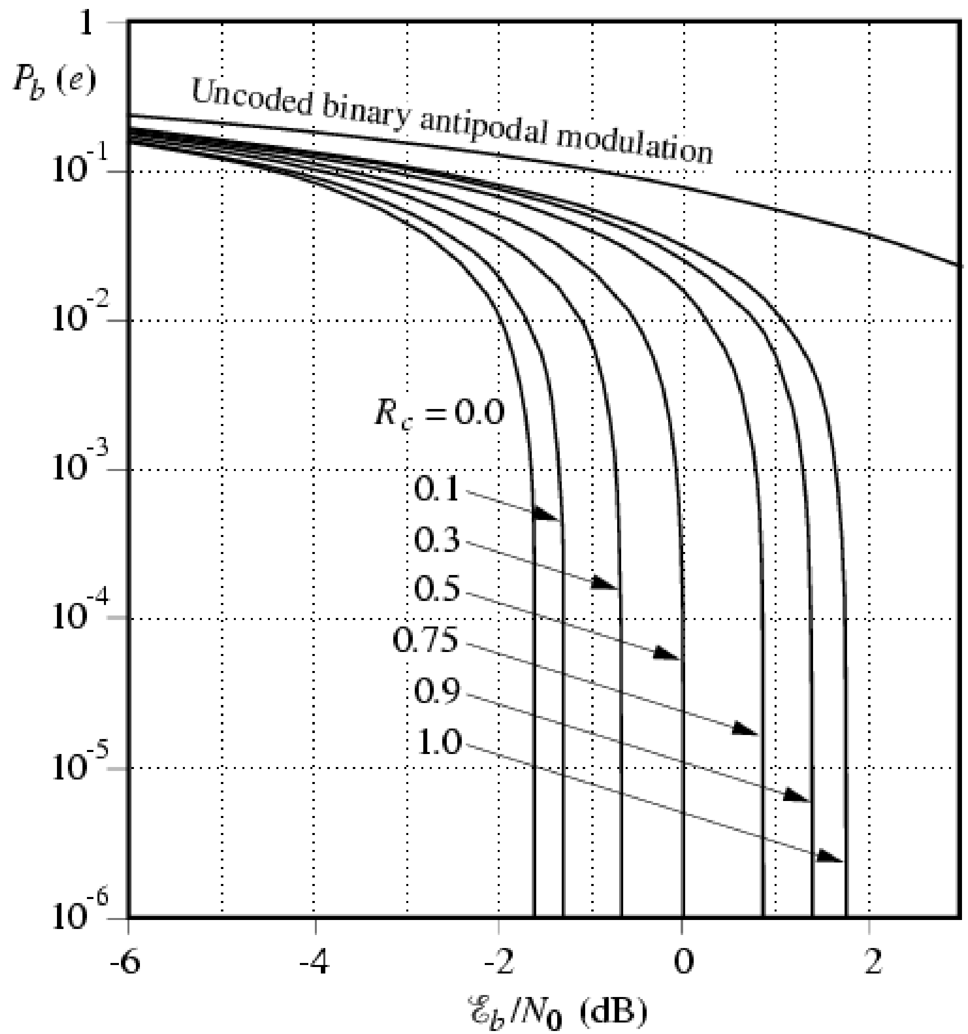
The concept of coding gain

- Curve A is the bit error probability versus SNR for uncoded binary antipodal modulation
- Curve B is the best we can do (Shannon converse theorem) over unconstrained AWGN channels
- Curve C is the best we can do (Shannon converse theorem) over binary symmetric channels



The concept of coding gain

- The maximum obtainable coding gain depends on the rate of the code, which in turns defines, for a given modulation, the spectral efficiency of the system
- The coding gain depends also on the desired bit error probability



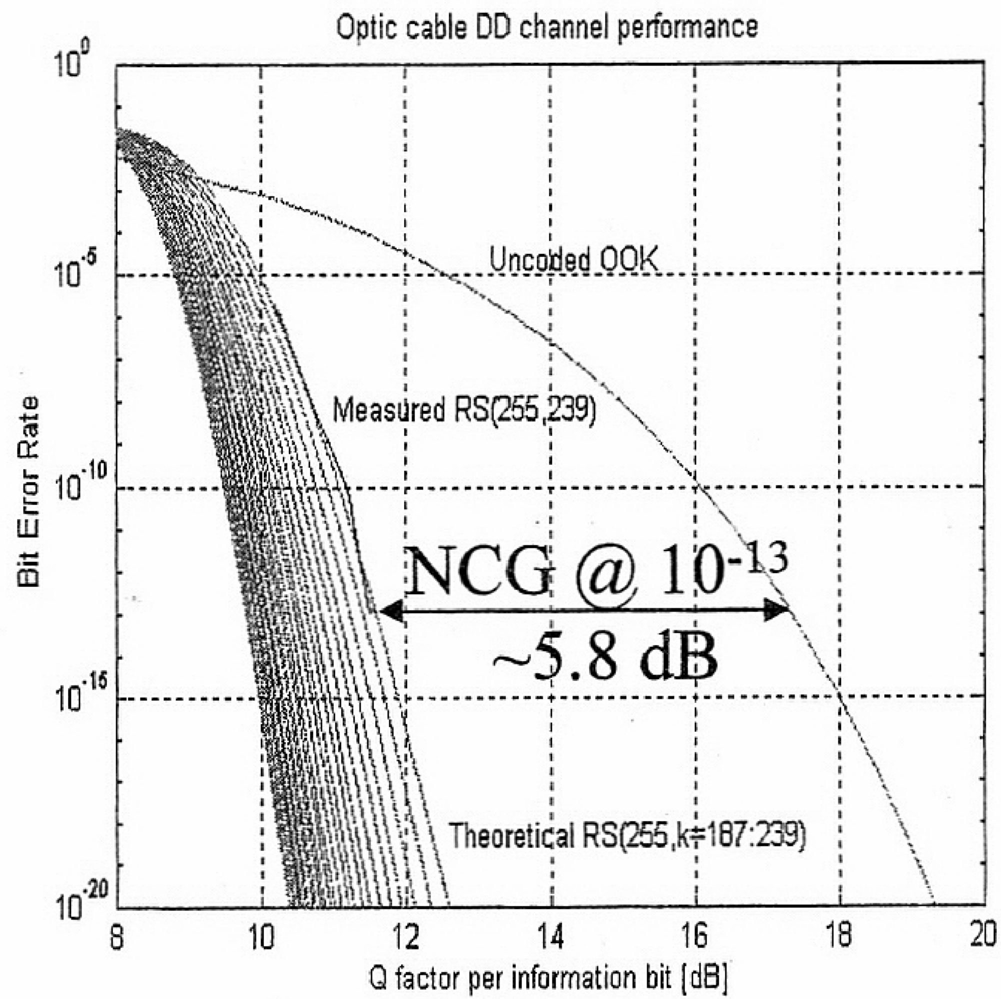
Coding for optical communications

Codes for optical communication should yield:

- Large coding gains (greater than 6 dB) with low complexity decoding
  - Concatenated algebraic codes with large block sizes
- Very low bit error probabilities (10^{-12} - 10^{-15})
  - Large minimum distance (very low "error floor") $\longrightarrow$ algebraic codes with large block sizes
- High code rates (overhead lower than 25%)
  - Block codes
- Very high information rates (up to 40 Gbit/s)
  - Low decoding complexity, hard or "quasi-hard"
  - Data flow demultiplexing or very fast hardware (up to 40 Gbit/s chip, Song et al., *IEEE J. of Solid State*, Nov. 2002)

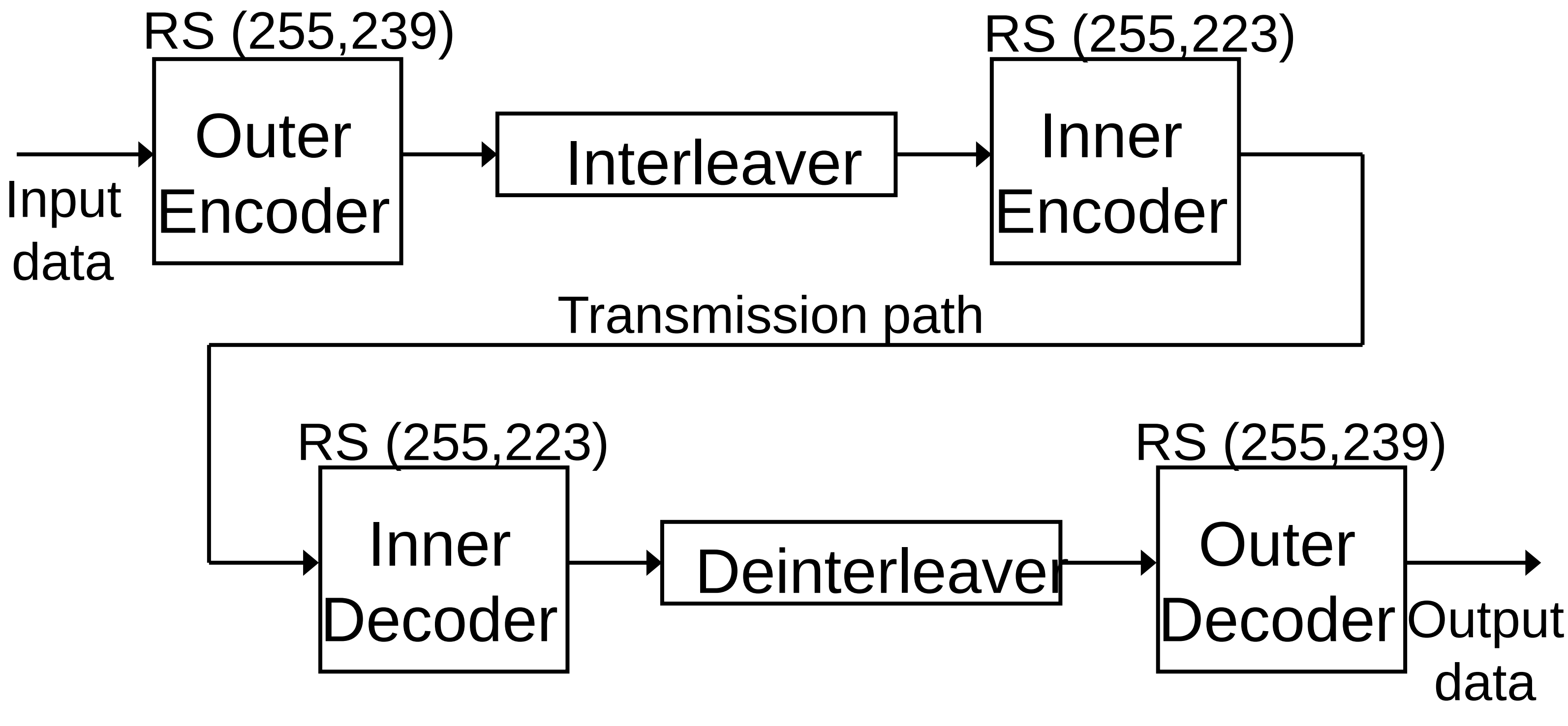
The "standard" coding scheme and its avatars

- ITU G.975 and ITU G.709 recommendations are based on Reed-Solomon codes, which are non-binary, systematic linear cyclic codes
- The RS (255,239) code was suggested, leading to a 6.7% overhead
- With hard decoding, a coding gain of 5.8 dB at bit error probability 10^{-13} is achievable



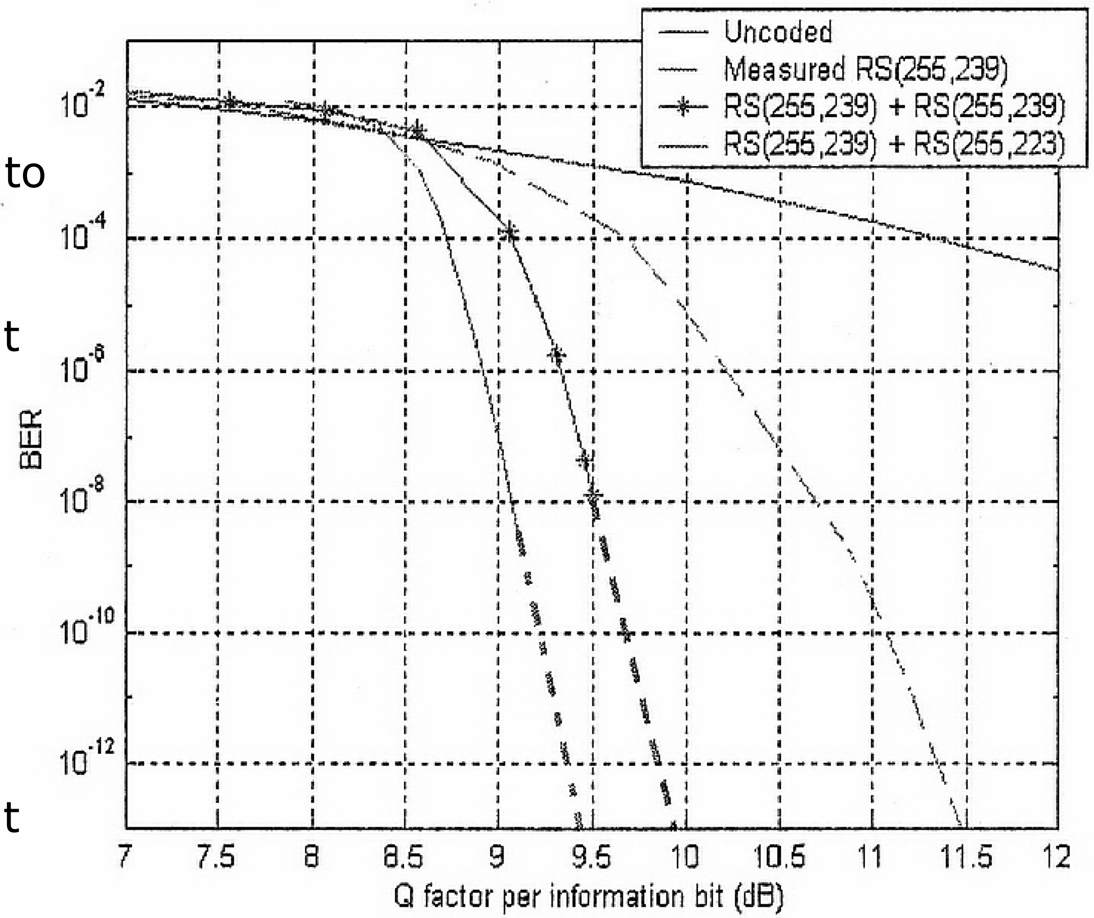
The "standard" coding scheme and its avatars

- To increase the coding gain, a solution based on the concatenation of two RS codes with hard decoding has been proposed



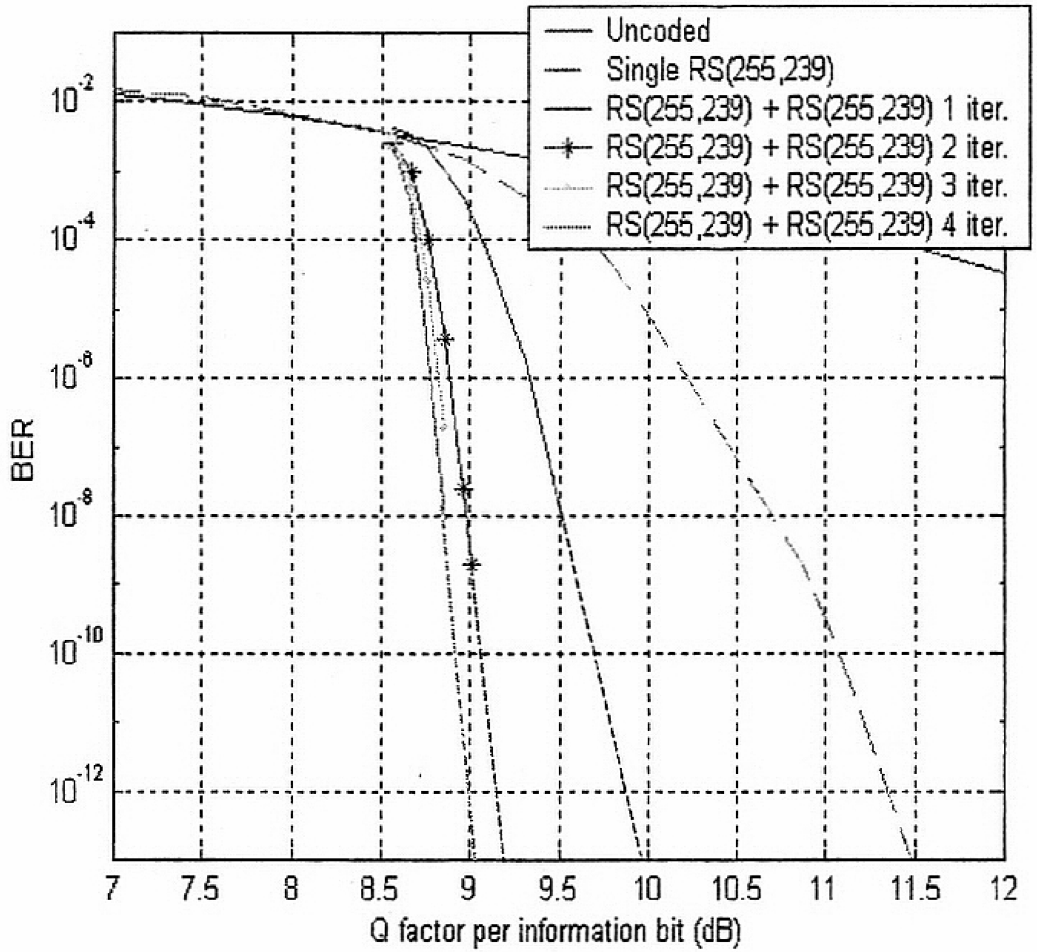
The "standard" coding scheme and its avatars

- The concatenation of two RS (255,239) codes leads to a 13.8% overhead
- With hard decoding, a coding gain of 7.4 dB at bit error probability 10^{-13} is achievable
- The concatenation of two RS codes, the outer a RS (255,239) and the inner a RS (255,223) leads to a 22% overhead
- With hard decoding, a coding gain of 7.9 dB at bit error probability 10^{-13} is achievable



The "standard" coding scheme and its avatars

- "One-shot" hard decoding is not the optimum way to decode a concatenated code
- Iterating several times the decoding algorithm, still based on hard samples, yields a further improvement
- The concatenation of two RS (255,239) codes (13.8% overhead) with iterative hard decoding yields a coding gain of 8.3 dB with 4 iterations
- No scope to increase the number of iterations beyond 4



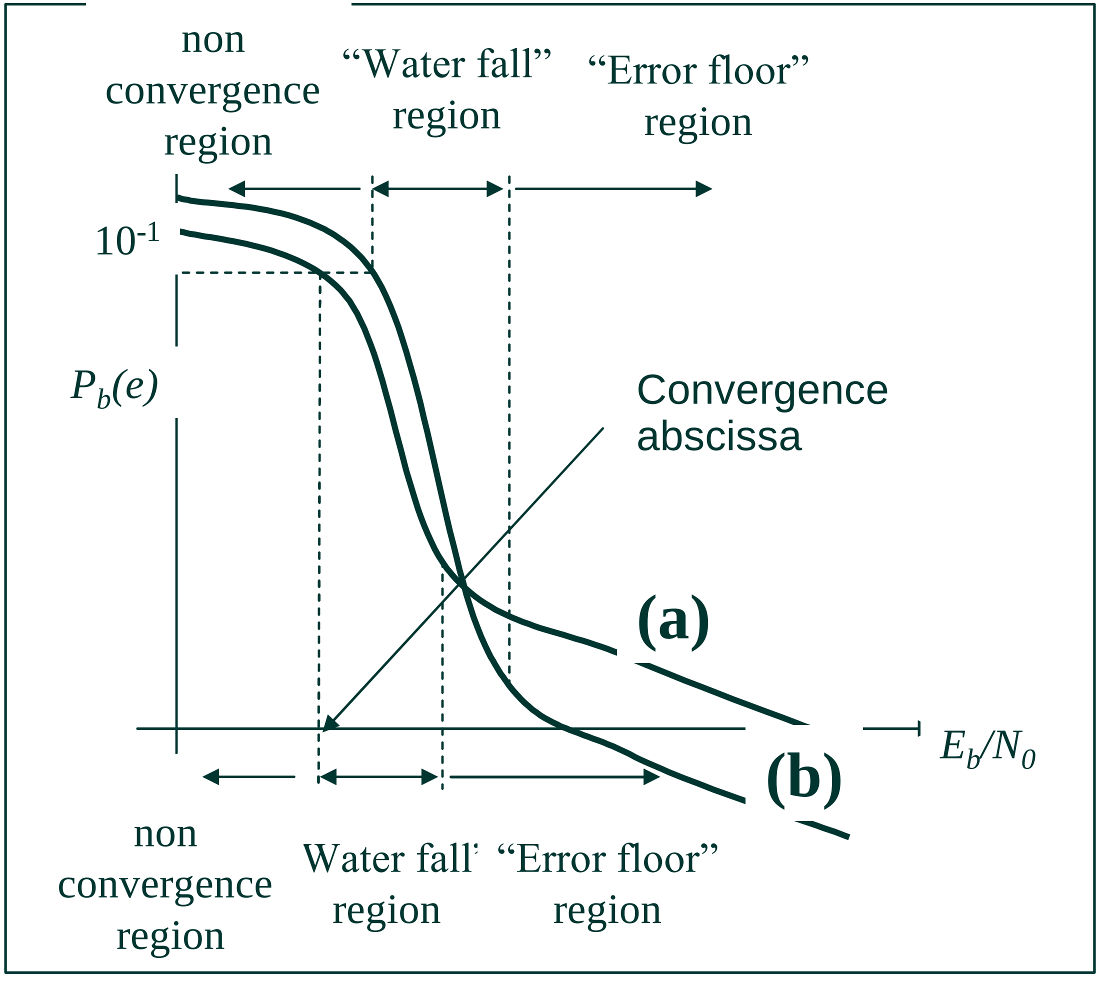
The impact of soft iterative decoding

- Soft versus hard decoding yields an increased coding gain of about 2 dB
- Soft decoding has almost the same complexity as hard decoding for convolutional codes (the celebrated Viterbi algorithm)
- For algebraic block codes, soft decoding is much more complex than hard decoding
- Soft decoding of RS codes is an active research field; the proposed solutions, though, are still too complex for optical communication
- We will describe two promising alternative schemes, based on turbo product codes and low-density parity-check codes

The impact of soft iterative decoding

- Three distinct regions of the bit error probability curves versus signal-to-noise ratio: the non-convergence, *waterfall* and *error floor* regions

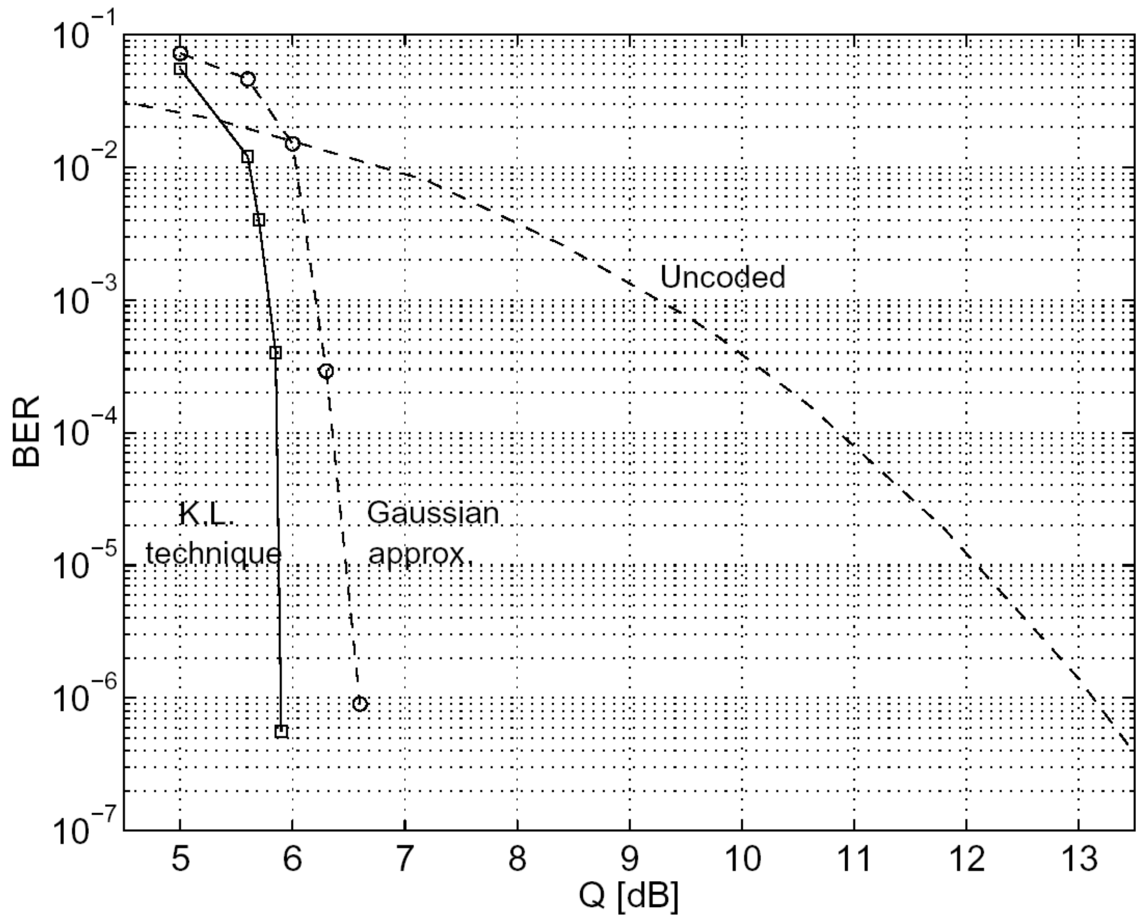
- The position of the error floor can be estimated by simulation (too complex at bit error probabilities below 10^{-12}), or by evaluating the minimum distance of the code and then analytical bounds



The impact of soft iterative decoding

- The effect of the Gaussian approximation on the log-likelihood ratios evaluation

- Continuous curve refers to the LLR evaluation using the Karhunen-Loève technique to model the optical communication channel

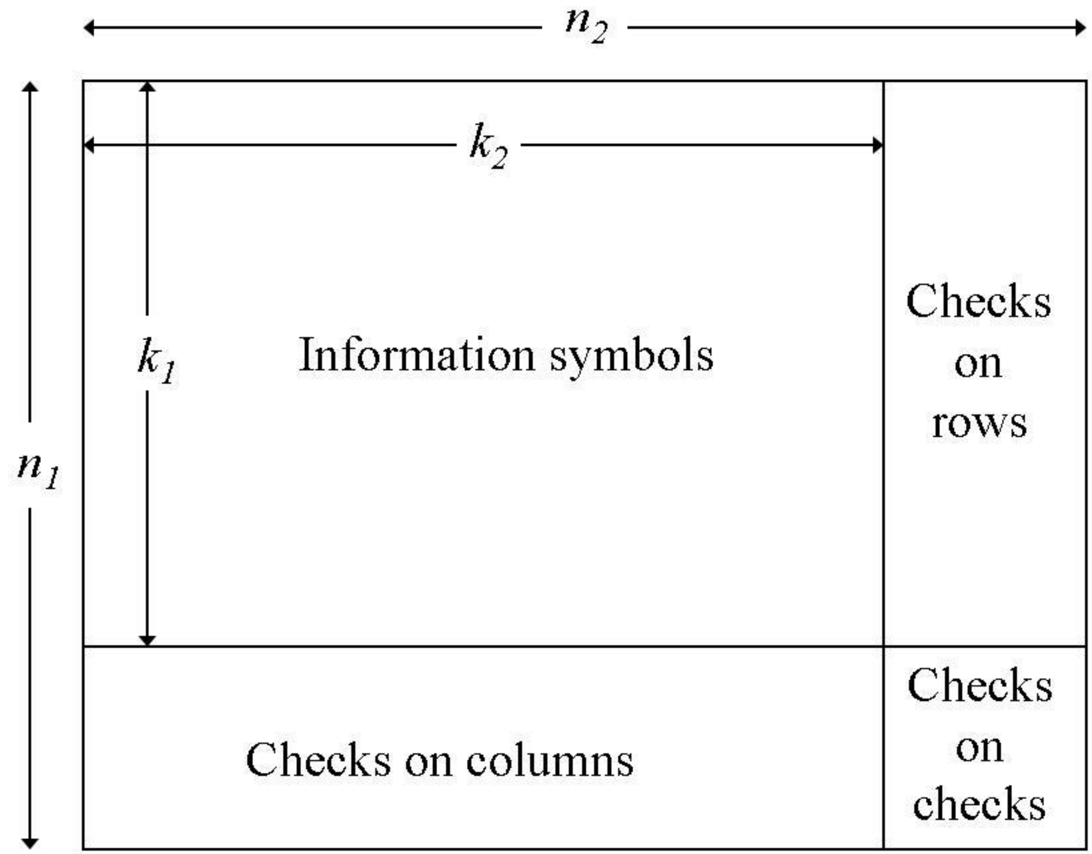


Turbo product codes

- Turbo product codes are serially concatenated block codes with interleaver
- The concatenated code parameters are:

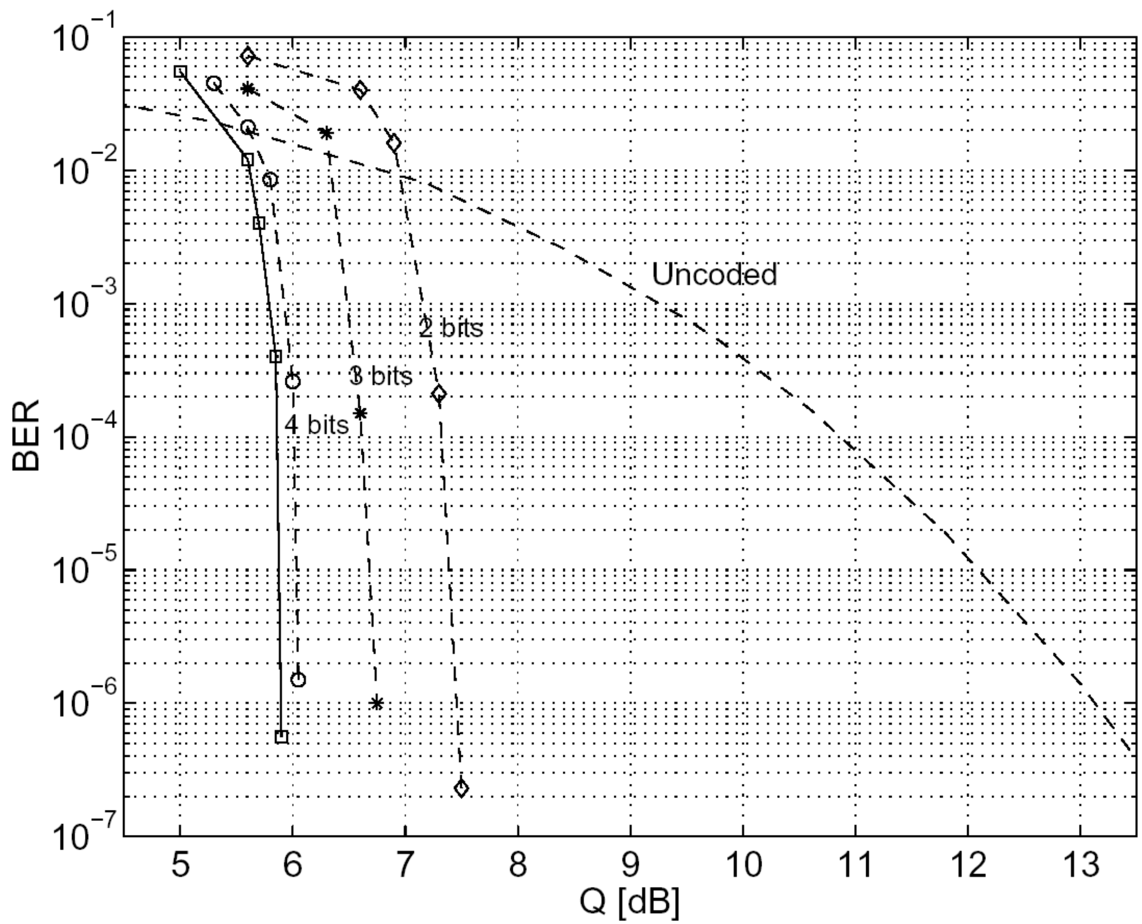
$$r_c = \frac{k_1 \times k_2}{n_1 \times n_2}$$

$$d_{\min} = d_{\min,1} + d_{\min,2}$$



Turbo product codes

- Turbo product code based on two (128,113) extended BCH codes, with minimum distance of 12
- Overhead is 28%, and (extrapolated) coding gain is 11.3 dB at bit error probability 10^{-13}
- The curves also show the effect of LLR quantization with different number of bits



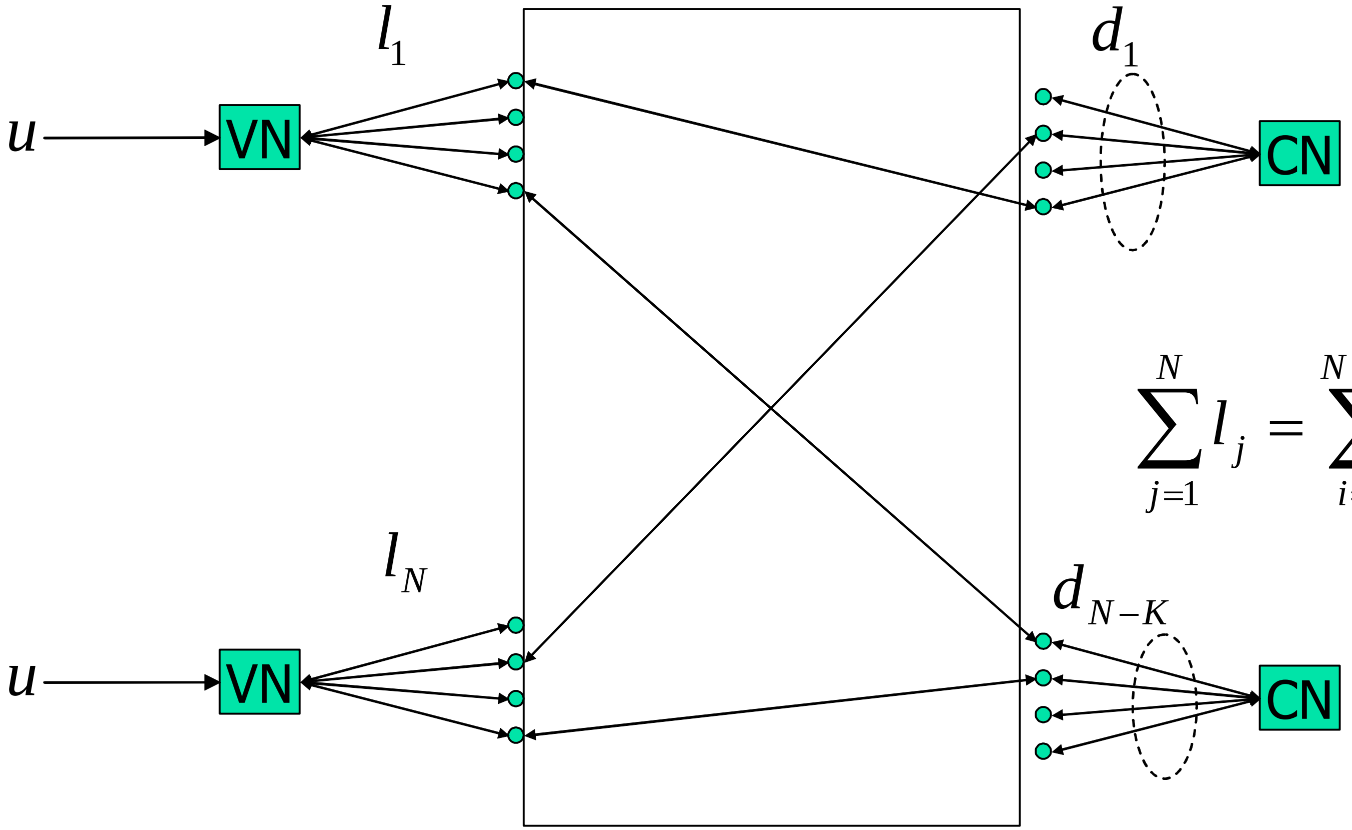
Turbo product codes

- State of the art in the use of block turbo codes is the experimental demonstration of a coding gain of 10.1 dB at bit error probability 10^{-13} using a code with 21% overhead and 3-bit soft decision at a data rate of 12.4 Gbit/s (T. Mizuochi et al., *OFC 2003*, March 2003)

Low-density parity-check codes

- Proposed by Gallager in 1962, and almost forgotten for 3 decades
- Deeply investigated after the invention of turbo codes in 1993
- LDPC codes are binary, linear block codes with a highly sparse parity-check matrix
- They can be *regular* (number of ones equal in all rows and column of the matrix), or *irregular* (they perform better than regular)
- Encoding complexity is linear with the block size
- Decoding is based on the *message* passing algorithm, a highly decentralized, iterative algorithm based on the repetition of simple computations in every node of the bipartite graph representing the encoder

Low-density parity-check codes

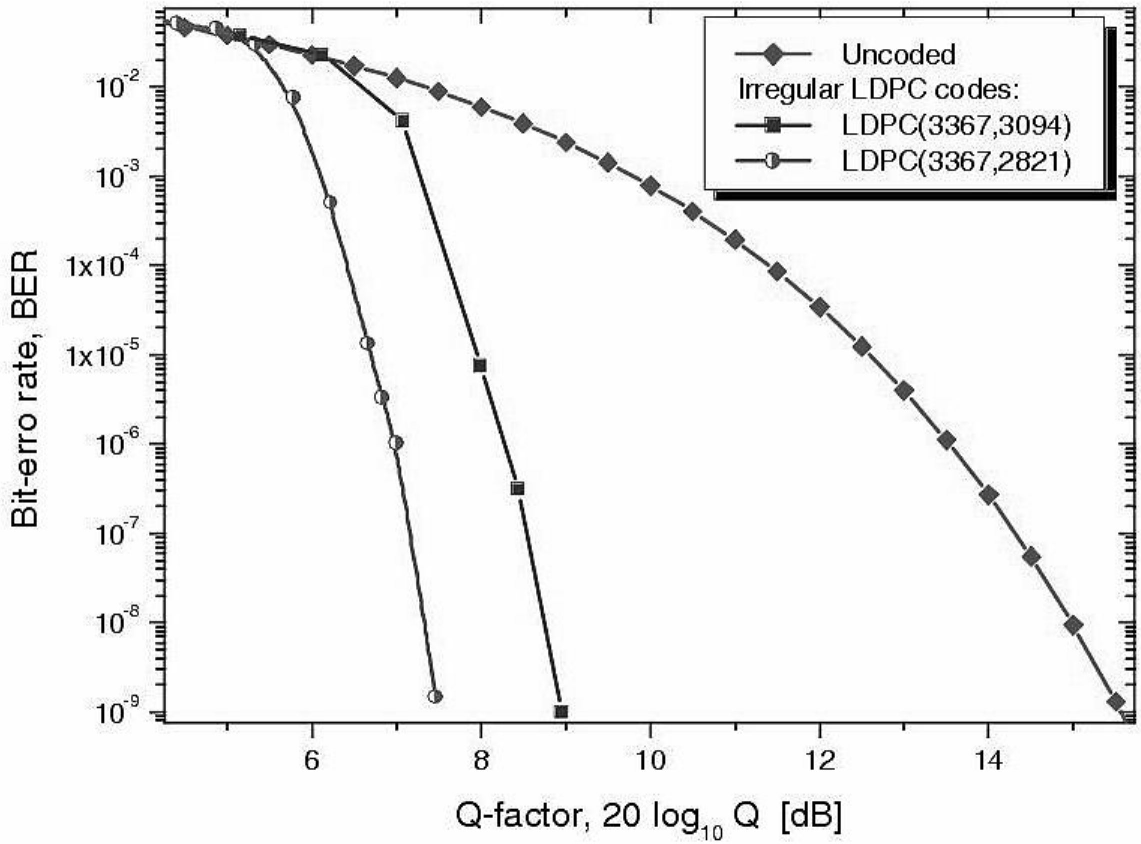


$$\sum_{j=1}^{N} l_j = \sum_{i=1}^{N-K} d_i$$

Low-density parity-check codes

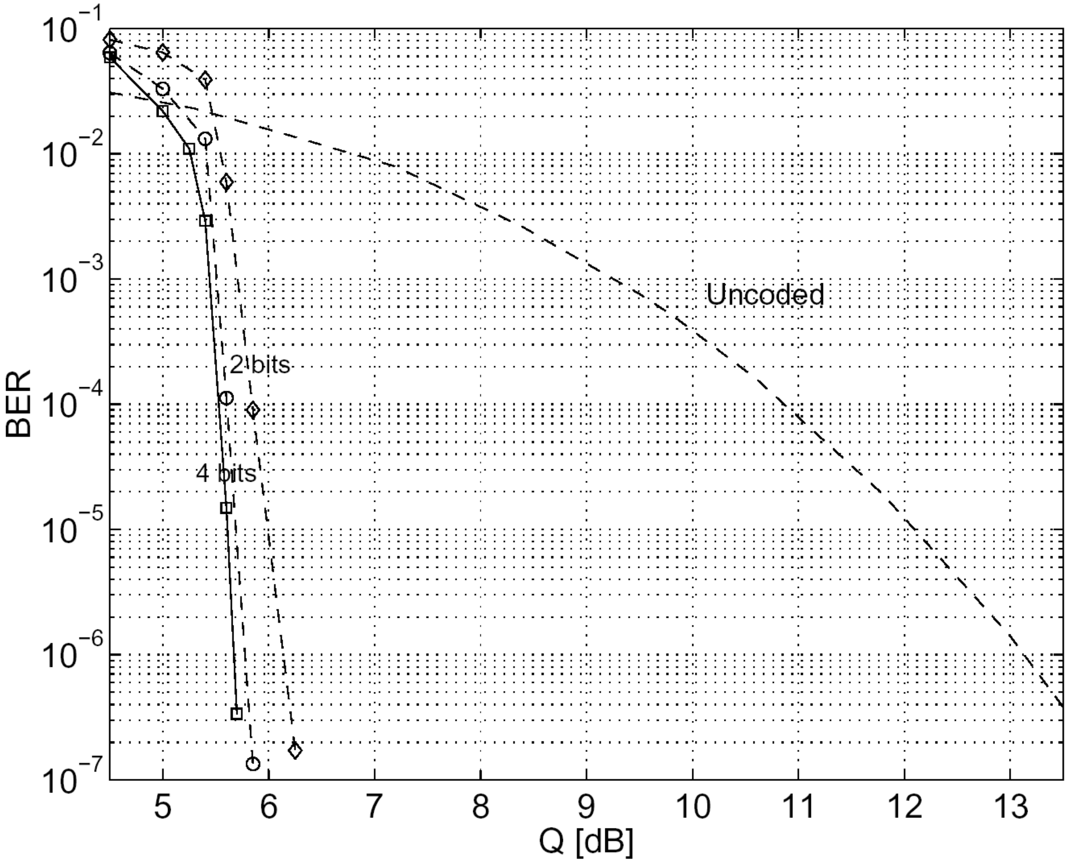
- Performance of two irregular LDPC codes
- LDPC (3367, 3094) has an overhead of 8.8%, with a coding gain of 6.7 dB at bit error probability of 10^{-9}
- LDPC (3367, 2821) has an overhead of 19.3%, with a coding gain of 8.1 dB at bit error probability of 10^{-9}

  (I. B. Djordjevic et al., *OFC 2004*)



Low-density parity-check codes

- Effect of quantization on LDPC decoders
- LDPC (3276, 2556) has an overhead of 28.1%, with a coding gain of 8.5 dB at bit error probability of 10^{-7}
- LDPC message-passing decoders are more robust than product turbo decoders
  (G. Bosco and
  S. Benedetto, *TIWDC 2004*)



High-speed parallel decoder architectures

- RS hard decoders working at data rates as high as 40 Gbit/s have already appeared

- The design of very high-speed iterative decoders requires decoding architectures with a large degree of parallelism

- LDPC message-passing decoders are ideal for parallel implementation, provided that the "collision" problem arising in writing into/reading from the common memory is solved
  - One possibility is to use LDPC encoders whose parity-check matrix has been constrained to be collision-free
  - A second, more general approach consists in reworking the addressing strategy in such a way that ***every*** code can be made collision-free (A. Tarable et al., *IEEE Transactions on Inf. Theory,* Sept. 2004)

- The decoder complexity stemming from the large number of iterations required by the message-passing algorithm can be reduced through the proper use of stopping criteria and a small amount of extra memory

Conclusions

- Constrained to hard, non-iterative decoding, the achievable coding gain for optical communications seems limited to roughly 8 dB with overheads in the order of 22%

- Use of soft decoding and iterative decoding algorithms can increase the coding gain up to more than 10 dB with the same overhead, BUT this requires:
  - Very high speed A/D converters, with 2-4 bits of precision
  - Highly parallel decoder architectures, with significant complexity
  - Unless fast HW is available, mixed simulation-analytical approaches to estimate the coding at very low bit error probabilities. In particular, the evaluation of the code minimum distance is required, a problem that is in general NP-complete