System impact of Sideband Instability

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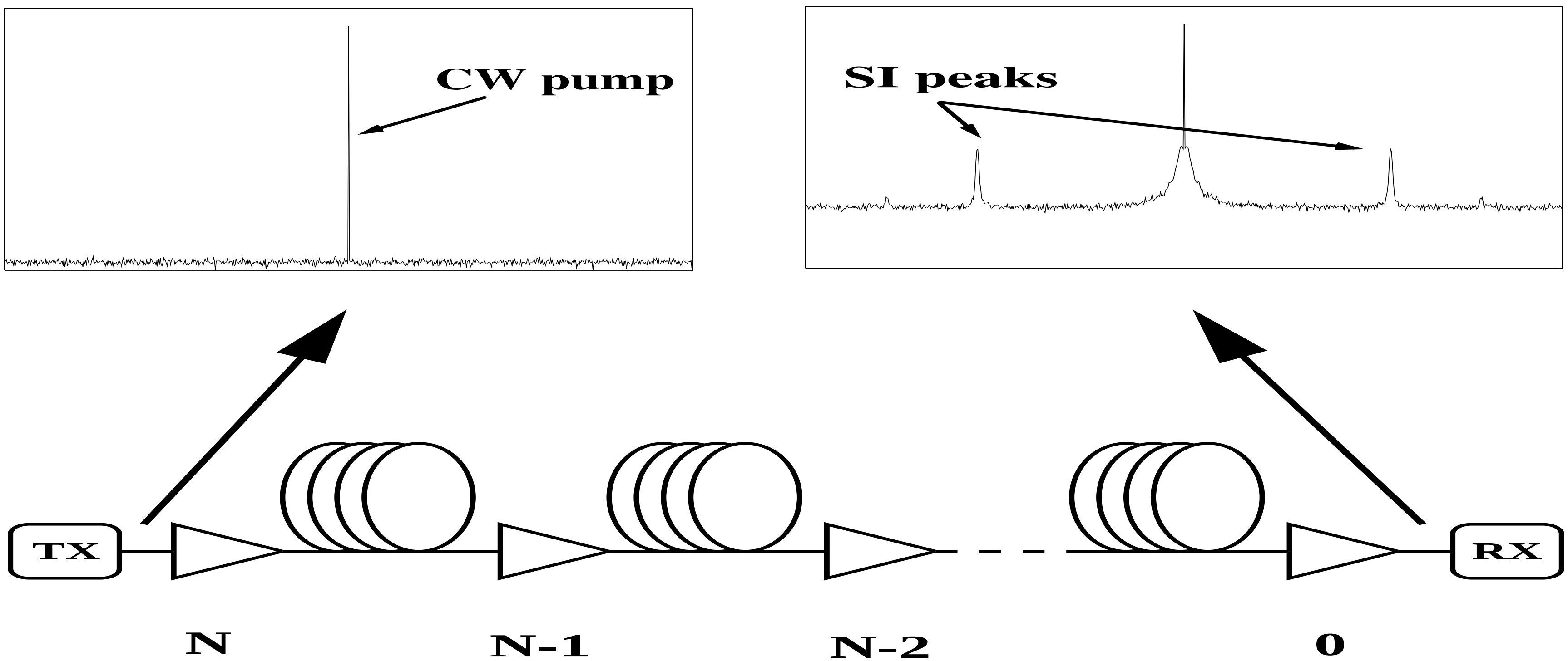
Outline

- What is Sideband Instability ?
- Theoretical background : Parametric Gain.
- SI analytical development.
- SI in real systems.
- Experimental results.
- Conclusions.

What is Sideband Instability ?



Sideband Instability: growth of sharp peaks in the output spectrum noise of an optically repeatered system.

The origin of Parametric Gain

- PG is caused by the interaction of fiber nonlinearities with dispersion.
- In both dispersion regimes, PG induces a transfer of optical power from the signal to the ASE noise in the adjacent spectral region.
- PG effects

Anomalous dispersion regime	$\Rightarrow$	Noise Enhancement
		Modulation Instability
Normal dispersion regime	$\Rightarrow$	Noise Enhancement

Single span PG scalar analysis

- Single polarization Nonlinear Schröedinger Equation (NLSE):

$$\frac{\partial U}{\partial z} = -\alpha U + \jmath \frac{1}{2} \beta_2 \frac{\partial^2 U}{\partial T^2} - \jmath \gamma \left| U \right|^2 U$$

- The signal is assumed to be of the form:

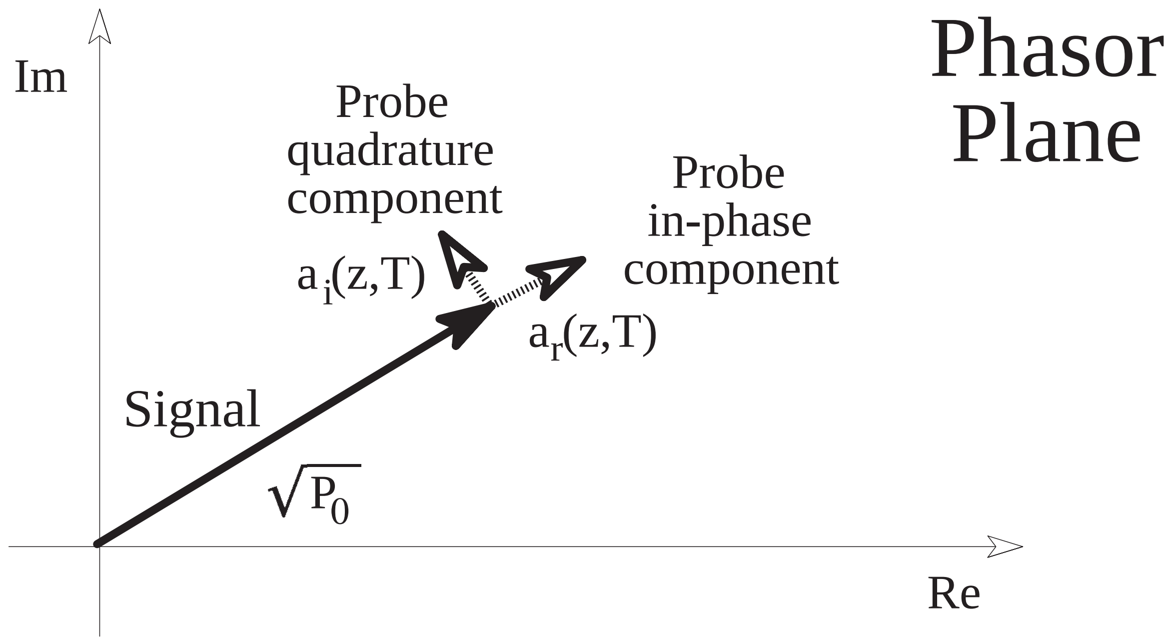
$$U(z, T) = \left[\sqrt{P_0} + a(z, T) \right] \mathrm{e}^{[-\alpha z + \jmath(\omega_0 T - \Phi_{NL})]}$$

where P_0 is the power of the pump signal, $a(z, T)$ is the small signal (probe), α is the fiber loss coefficient, γ is the nonlinearity coefficient and

$$\Phi_{NL} = \gamma P_0 \int_0^z \mathrm{e}^{(-2\alpha\xi)}\, \mathrm{d}\xi$$

is the phase-shift induced by fiber nonlinearities.

Signal and probe representation



Probe components

time domain

$$a_r(z,t) = \mathcal{R}e\,\{a(z,T)\}$$

$$a_i(z,t) = \mathcal{I}m\,\{a(z,T)\}$$

Probe vector

frequency domain

$$\underline{\Theta}(z,\Omega) = \begin{bmatrix} A_r(z,\Omega) \\ A_i(z,\Omega) \end{bmatrix} = \begin{bmatrix} \mathcal{F}\{a_r(z,t)\} \\ \mathcal{F}\{a_i(z,t)\} \end{bmatrix}$$

PG influence on ASE noise

- Transfer matrix formalism

$$\underline{\Theta}(z,\Omega) = \underline{\underline{T}}\ \underline{\Theta}(0,\Omega)$$

- Spectrum Matrix of a vectorial 2-dimensions random process

$$\underline{\underline{\mathcal{G}}}(z,\Omega) = \begin{bmatrix} \mathcal{G}_{rr}(z,\Omega) & \mathcal{G}_{ri}(z,\Omega) \\ \mathcal{G}_{ir}(z,\Omega) & \mathcal{G}_{ii}(z,\Omega) \end{bmatrix} = \begin{bmatrix} \mathcal{F}\{R_{a_r a_r}(z,\tau)\} & \mathcal{F}\{R_{a_r a_i}(z,\tau)\} \\ \mathcal{F}\{R_{a_i a_r}(z,\tau)\} & \mathcal{F}\{R_{a_i a_i}(z,\tau)\} \end{bmatrix}$$

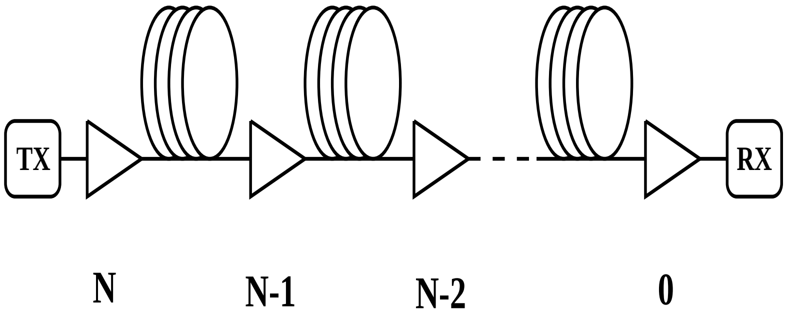
- Evolution of the Spectrum Matrix through a linear system:

$$\underline{\underline{\mathcal{G}}}(z,\Omega) = \underline{\underline{T}}(z,\Omega) \cdot \underline{\underline{\mathcal{G}}}(0,\Omega) \cdot \underline{\underline{T}}^{\dagger}(z,\Omega)$$

- PG action and normalization $\Longrightarrow$ PG noise gain matrix

$$\underline{\underline{G}}(z,\Omega) = \begin{bmatrix} |T_{11}|^2 + |T_{12}|^2 & T_{11}T_{21} + T_{12}T_{22} \\ T_{11}T_{21} + T_{12}T_{22} & |T_{21}|^2 + |T_{22}|^2 \end{bmatrix}$$

Sideband Instability: theoretical analysis (I)



Hypotheses:

- Neglecting signal depletion
- EDFA recover span loss
- Perfect periodicity

Multispan periodic link

$$\underline{\Theta}_{out}(\Omega) = \sum_{i=0}^{N} \underline{\underline{T}}^i \underline{\Theta}_{in,i}(\Omega)$$

$\underline{\Theta}_{in,i}(\Omega)$ noise added by the i-th EDFA

$$\underline{\Theta}_{in,i}(\Omega) = \underline{\Theta}_{in}(\Omega) \quad \forall i = 0, \ldots, N$$

Equivalent transfer matrix

$$\underline{\Theta}_{out}(\Omega) = \underline{\underline{T}}^{(N)} \underline{\Theta}_{in}(\Omega)$$

$$\underline{\underline{T}}^{(N)} = \sum_{i=0}^{N} \underline{\underline{T}}^i$$

Sideband Instability: theoretical analysis (II)

- **Noise gain matrix of a multispan periodic link**

$$\underline{\underline{G}}_{out}(\Omega) = \begin{bmatrix} |T_{11}^{(N)}|^2 + |T_{12}^{(N)}|^2 & T_{11}^{(N)} T_{21}^{(N)} + T_{12}^{(N)} T_{22}^{(N)} \\ T_{11}^{(N)} T_{21}^{(N)} + T_{12}^{(N)} T_{22}^{(N)} & |T_{21}^{(N)}|^2 + |T_{22}^{(N)}|^2 \end{bmatrix}$$

- **Sylvester's theorem: an analytical expression for $\underline{\underline{T}}^k$**

$$\underline{\underline{T}}^k = \begin{bmatrix} T_{11} Q_{k-1} - Q_{k-2} & T_{12} Q_{k-1} \\ T_{21} Q_{k-1} & T_{22} Q_{k-1} - Q_{k-2} \end{bmatrix}$$

where

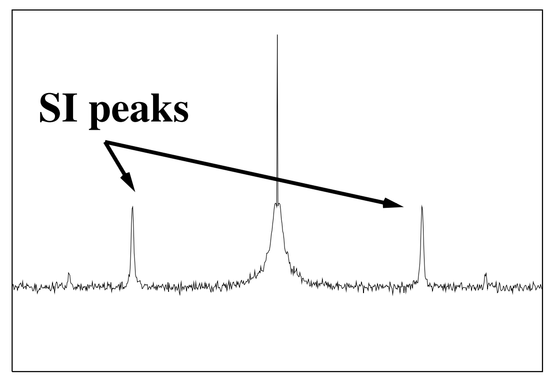
$$Q_{k-i} = \frac{\sin((k-i-1)\theta)}{(k-i-1)\theta} \text{ and } \theta = \arccos\left[\frac{T_{11} + T_{22}}{2}\right]$$

Sideband Instability: theoretical analysis (III)

Sideband Instability peaks condition

$$\left| \frac{T_{11}(\Omega) + T_{22}(\Omega)}{2} \right| > 1$$

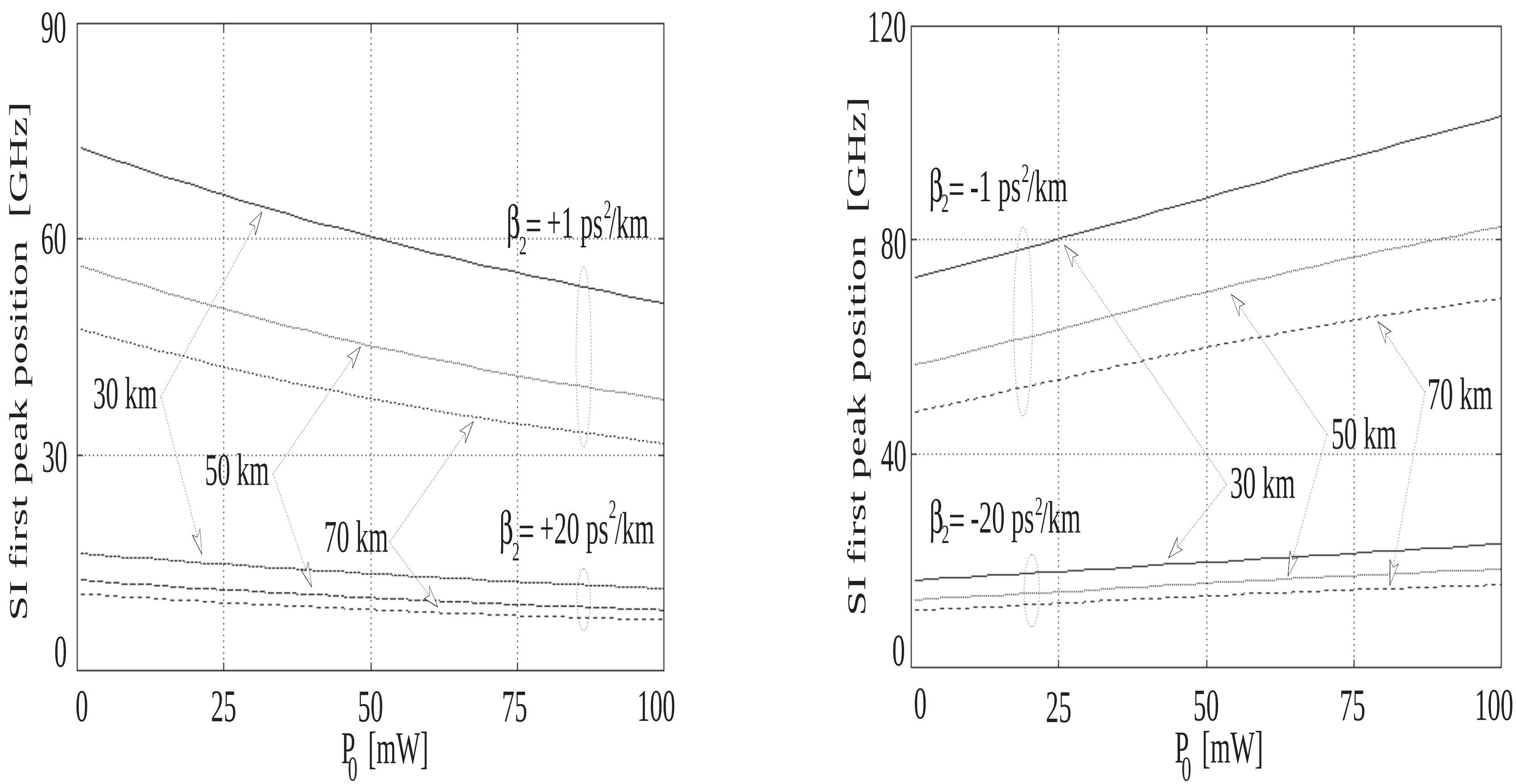
- In these spectral regions:
  - Q_{k-i} grow exponentially
  - $\underline{\underline{T}}_{ij}^{(N)}$'s follow this behaviour
  - $\underline{\underline{G}}_{out}$ elements present sharp peaks $\Rightarrow$ SI



SI peaks position versus system parameters (I)

- Using numerical investigation we found SI peaks position starting from a single span TM
- SI peaks of order higher than first are usually negligible
- SI peaks position depends on:
  - L, span length
  - β_2, dispersion parameter
  - γP_0, nonlinear parameter and pump power
- SI peaks position does not depend on:
  - N, number of span

SI peaks position versus system parameters (II)



Normal dispersion. **Anomalous dispersion.**

Sideband Instability in real systems (I)

SI needs a strong phase matching.

SI peaks are reduced by random variation of system parameters:

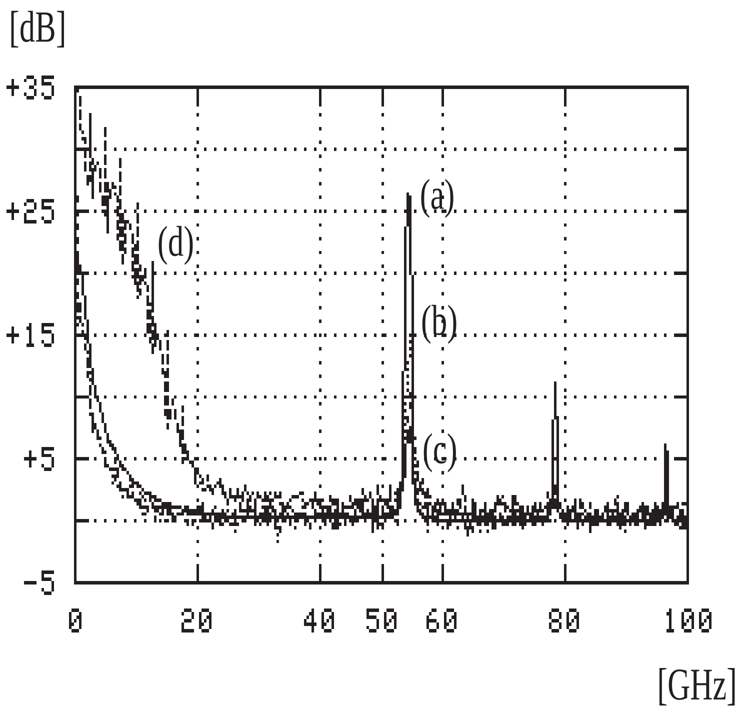
- Span length: its value changes around the expected value;
- Dispersion: its value changes around the expected value;
- Birefringence: it changes modulus and axes;
- PMD: like birefringence.

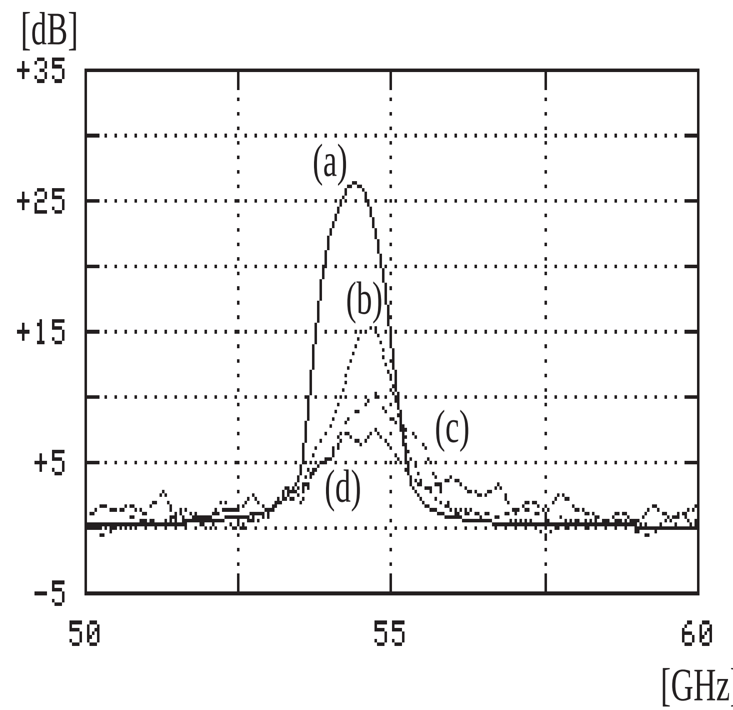
Pump depletion and non ideal EDFA behaviour also reduce SI peaks because break the periodicity.

Recirculating loops

Strict periodicity $\Rightarrow$ higher SI peaks

Sideband Instability in real systems (II)





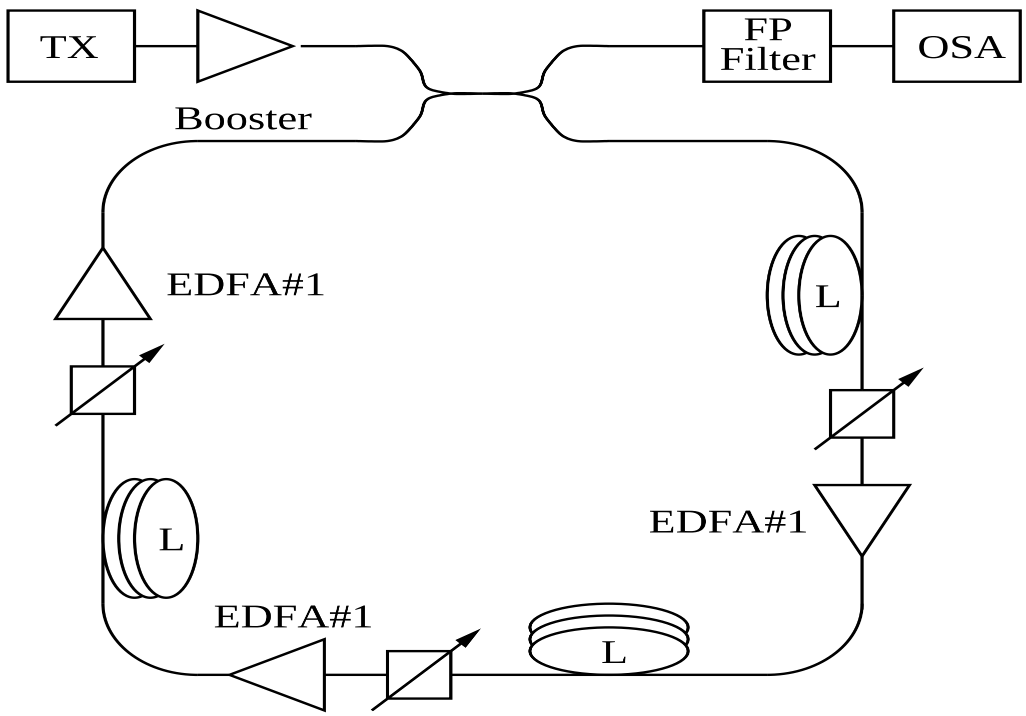
(a) scalar theoretical analysis

(b) vectorial simulation with random PMD and birefringence

(c) like (b) with random dithering of dispersion and span length

(d) like (c) with 10 MHz linewidth pump signal modulated at 2.5 Gbit/s

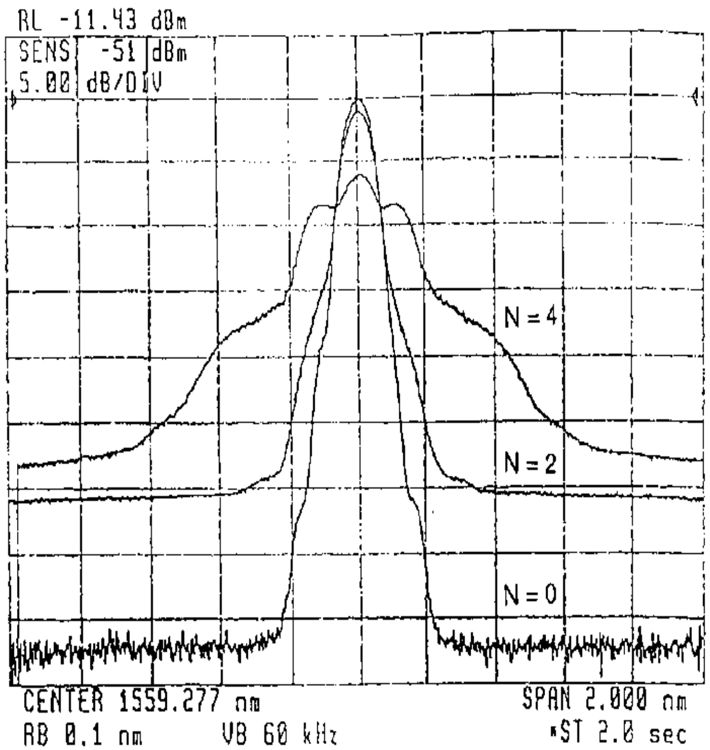
SI evidence in a recirculating loop experimental set-up



- R_b = 2.5 Gbit/s
- $\overline{P}_0$= +9 dBm
- D = ± 1. ps/nm/km
- α = 0.22 dB/km
- L = 80 km

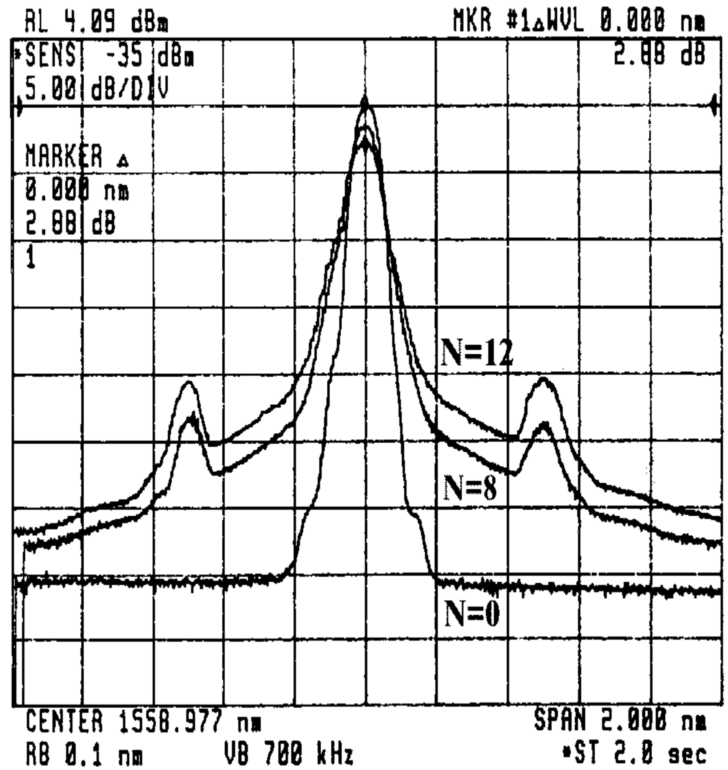
Experiment carried out at Pirelli Cables & Systems laboratories.

Experimental results



Anomalous dispersion.

D = + 1 ps^2/km



Normal dispersion.

D = - 1 ps^2/km

Conclusions

- In real system SI impact is usually negligible.
- Recirculating loops are affected by strong SI: from this point of view they may be non-realistic test-beds for long-haul systems under particular conditions.
- The formalism we developed can predict SI peaks positions allowing to better understand measurements on recirculating loops.

Acknowledgment

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