



Modelling and mitigation of fibre nonlinearity in WDM coherent optical systems

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Outline



- **Modelling of fibre nonlinearity**
 - Modelling approximations
 - The GN/EGN model family
- **Mitigation of fibre nonlinearity**
 - Theoretical limits
 - Practical performance limits



MODELLING OF FIBRE NONLINEARITY

Non-linear fibre propagation models



- Any form of analytical description of the non-linear behaviour of the optical fibre
- Example: coupled non-linear Schrödinger equations

$$\begin{aligned} \frac{\partial A_x}{\partial z} + \beta_{1x} \frac{\partial A_x}{\partial t} + \frac{i\beta_2}{2} \frac{\partial^2 A_x}{\partial t^2} + \frac{\alpha}{2} A_x &= i\gamma(|A_x|^2 + B|A_y|^2)A_x \\ \frac{\partial A_y}{\partial z} + \beta_{1y} \frac{\partial A_y}{\partial t} + \frac{i\beta_2}{2} \frac{\partial^2 A_y}{\partial t^2} + \frac{\alpha}{2} A_y &= i\gamma(|A_y|^2 + B|A_x|^2)A_y \end{aligned}$$

G. P. Agrawal, *Nonlinear Fiber Optics*, 4th edition. Academic Press, 2007, Chapter 6.

- Numerical integration within a Monte-Carlo simulation environment
- Goal: to find simpler yet accurate models in order to quantify the system impact of the fibre non-linear behaviour

First approximation: the Manakov equation



$$\begin{cases} \frac{\partial A_x(z,t)}{\partial z} = j \frac{\beta_2}{2} \frac{\partial^2}{\partial t^2} A_x(z,t) - \alpha A_x(z,t) - j\gamma \frac{8}{9} \left[|A_x(z,t)|^2 + |A_y(z,t)|^2 \right] A_x(z,t) \\ \frac{\partial A_y(z,t)}{\partial z} = j \frac{\beta_2}{2} \frac{\partial^2}{\partial t^2} A_y(z,t) - \alpha A_y(z,t) - j\gamma \frac{8}{9} \left[|A_x(z,t)|^2 + |A_y(z,t)|^2 \right] A_y(z,t) \end{cases}$$

- It's based on an analytical average over the random evolution of the state-of-polarization (SOP) along the fibre
- It captures the non-linear effects of one polarization onto the other, but averages over the fast dynamic of SOP variations
- It neglects both linear and nonlinear effects of PMD

Families of models



- Examples:
 - time domain
 - frequency domain
 - Volterra-based
 - first order perturbation
 - higher-order perturbation
 - regular perturbation (RP, with variants)
 - logarithmic perturbation (LP, with variants)
 - pulse-collision based
 - more classes and sub-classes based on specific assumptions and approximations...
- In this talk, I will focus on **frequency-domain RP first-order models**



Modelling approximations

- Perturbation approach
- NLI as additive Gaussian noise
- Locally white NLI
- Signal Gaussianity
- Incoherent NLI accumulation
- Single-polarization
- Lossless fibre
- Noiseless propagation
-

First-order regular perturbation



- Assumptions:
 - The signal propagates linearly from input to output
 - At each point along the fibre, it excites fibre nonlinearity and creates the NLI disturbance
 - At the end of the fibre, the linearly propagated signal and the NLI are summed (**NLI noise can be represented as an additive noise term**)

$$s_{WDM}^{NL}(t) = s_{WDM}(t) + s_{NLI}(t) \quad \text{NON-LINEAR INTERFERENCE (NLI)}$$

- In the framework of first-order perturbation analyses, the NLI power is proportional to P_{ch}^3 :

$$P_{NLI} = \eta P_{ch}^3$$

- where η is a coefficient that depends on the fibre parameters and the transmitted signal characteristics.



NLI additive Gaussian noise approximation

- Assumption:
 - the NLI at the output of the link can be represented as additive Gaussian noise, circular and independent of either the signal or ASE noise
- Key implication: the channel performance can be characterized based on a modified “non-linear” OSNR:

$$OSNR_{NL} = \frac{P_{ch}}{P_{ASE} + P_{NLI}}$$

- P_{ch} : power of channel under test
- P_{ASE} : power of ASE noise (e.g. $P_{ASE} = G_{ASE}(f_c)R_s = Fl\nu(G-1)N_{span}R_s$ with EDFA amp.)
- P_{NLI} is the power of NLI

$$P_{NLI} = \frac{\int_{-\infty}^{+\infty} G_{NLI}(f+f_c) |H_{Rx}(f)|^2 df}{\int_{-\infty}^{+\infty} |H_{Rx}(f)|^2 df} R_s$$

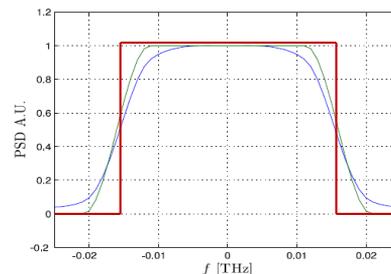
Locally white NLI noise approximation



- Assumption:
 - the PSD of NLI is locally flat (over a single channel bandwidth)

$$P_{NLI} \cong \frac{\int_{-\infty}^{+\infty} G_{NLI}(f_c) |H_{Rx}(f)|^2 df}{\int_{-\infty}^{+\infty} |H_{Rx}(f)|^2 df} R_s = G_{NLI}(f_c) R_s$$

$$OSNR_{NL} = \frac{P_{ch}}{[G_{ASE}(f_c) + G_{NLI}(f_c)] R_s}$$



Tx signal PSD
PSD of NLI
Approximated
PSD of NLI

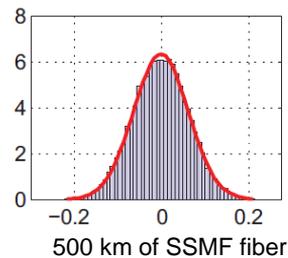
- This assumption is acceptable for approximate system performance assessment.
- It should be removed for high-accuracy predictions.



The signal Gaussianity approximation

- Assumption:

- the transmitted signal can be modelled as a stationary circular Gaussian noise, whose PSD is shaped as the PSD of the actually transmitted WDM channels.



- This approximation allows to drastically simplify the model derivation and strongly decreases the model final analytical complexity.
- Using this assumption, the impact of NLI is always overestimated for QAM transmission formats.



The incoherent NLI accumulation approximation

- Assumption:

- the NLI produced in each span adds up incoherently (i.e., in power) at the receiver site.

$$G_{NLI}(f) \approx \sum_{n=1}^{N_{span}} G_{NLI}^{(n)}(f)$$

- In reality, the NLI contributions should be added together coherently (i.e., at the field level) keeping both their amplitude and phase into account
- The accuracy of this approximation is quite poor at very low span count and at very low channel count.



The EGN-GN model family

Assumption	EGN model	GN model	iGN model
Manakov equation	X	X	X
1 st order regular perturbation	X	X	X
Signal Gaussianity		X	X
Incoherent NLI accumulation			X
NLI as additive Gaussian noise	<i>Approximations that can be applied to all models in order to simplify the computations</i>		
Locally white NLI			

- **iGN** – P. Poggiolini et al., “Analytical Modeling of Nonlinear Propagation in Uncompensated Optical Transmission Links”, IEEE Photon. Technol. Lett. **23**(11), p. 742 (2011).
- **GN** – P. Poggiolini “The GN Model of Non-Linear Propagation in Uncompensated Coherent Optical Systems,” *J. Lightwave Technol.* **30**(24), p.3857 (2012).
- **EGN** – A. Carena et al., “EGN model of non-linear fiber propagation,” *Opt. Exp.* **22**(13), p. 16335, 2014.



The simplest iGN closed-form solution

- All approximations listed in the previous slide, plus ...
 - Equal spans
 - Equal channels (same power, same spectrum with bandwidth $\sim R_s$)

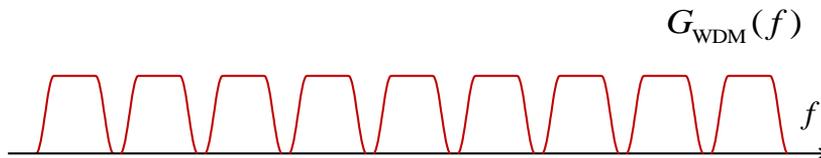
$$G_{\text{NLI}}(f_c) = N_{\text{span}} \frac{16}{27} \frac{\gamma^2 L_{\text{eff}}^2 P_{\text{ch}}^3}{\pi |\beta_2| \alpha R_s^3} \operatorname{asinh} \left(\frac{\pi^2}{2\alpha} |\beta_2| R_s^2 \left[N_{\text{ch}}^2 \right]^{\frac{R_s}{\Delta f}} \right)$$

- The model equations become more and more complex, as well as more and more accurate, as the various assumptions are removed



The GN-model reference formula

$$G_{\text{NLI}}(f) = \frac{16}{27} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} G_{\text{WDM}}(f_1) G_{\text{WDM}}(f_2) G_{\text{WDM}}(f_1 + f_2 - f) \cdot |\mu(f_1, f_2, f)|^2 df_1 df_2$$



**FWM efficiency
of the whole link**

- For identical spans with lumped amplification:

$$|\mu(f_1, f_2, f)|^2 = \gamma^2 L_{\text{eff}}^2 \left| \frac{1 - e^{-2\alpha L_s} e^{j4\pi^2 \beta_2 L_s (f_1 - f)(f_2 - f)}}{1 - j2\pi^2 \beta_2 \alpha^{-1} (f_1 - f)(f_2 - f)} \right|^2 \frac{\sin^2(2N_s \pi^2 (f_1 - f)(f_2 - f) \beta_2 L_s)}{\sin^2(2\pi^2 (f_1 - f)(f_2 - f) \beta_2 L_s)}$$

The EGN model



- EGN model** stands for “enhanced GN model”
- The EGN model consists of the GN model and of a “correction” term:

$$G_{\text{NLI}}^{\text{EGN}}(f) = G_{\text{NLI}}^{\text{GN}}(f) - G_{\text{NLI}}^{\text{corr}}(f)$$

- For PM-QAM systems the “correction” always decreases NLI
 - this shows the GN-model to be some sort of “upper bound” to NLI

Components of NLI



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Long-correlated phase and polarization rotation (PPRN) noise

Format-dependent

- Negligible for constant-modulus formats
- Increases with the cardinality of the constellation
- Maximum for Gaussian constellation
- Can be partially mitigated by CPE

GN-model

P_{NLI}



Short-correlated circular noise

Mainly format-independent

Components of NLI



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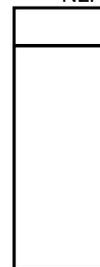
Long-correlated phase and polarization rotation (PPRN) noise

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GN-model

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EGN-model

Components of NLI

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Mainly format-independent

EGN-model

EGN-model
for QPSK
(EGN-cm model)

Components of NLI

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Long-correlated phase and polarization rotation (PPRN) noise

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GN-model

P_{NLI}

Short-correlated circular noise

Mainly format-independent

EGN-model

EGN-model
for QPSK
(EGN-cm model)

Practical implication



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If all the long-correlated phase noise is ideally taken out, then:

any PM-QAM system is well described by the EGN model, calculated as if PM-QPSK was transmitted (EGN-cm model)

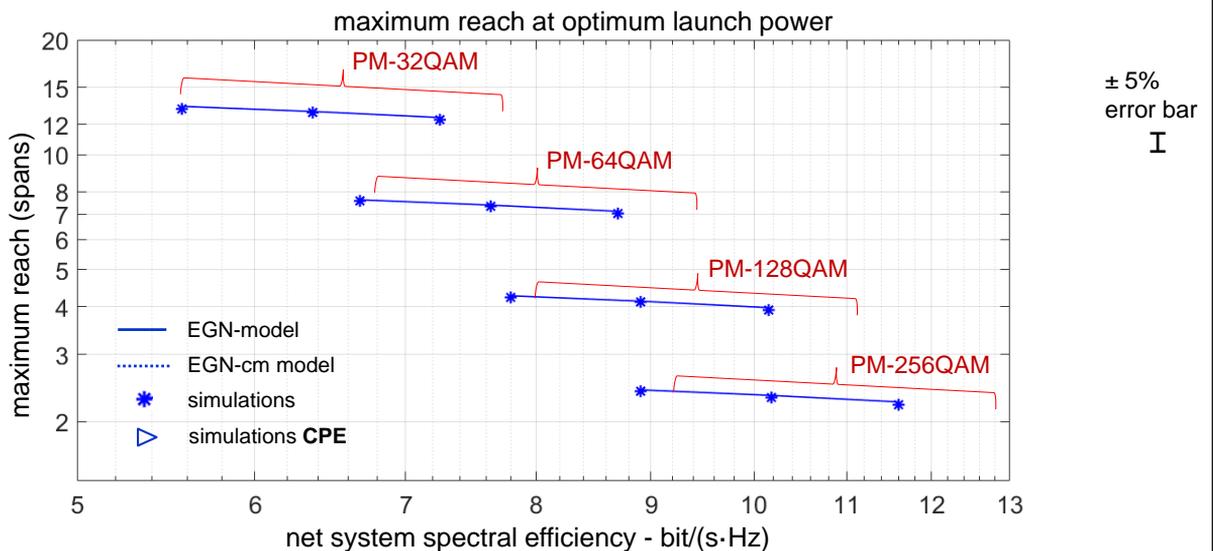
A. Nespola et al, "Independence of the Impact of Inter-Channel Non-Linear Effects on Modulation Format and System Implications," Proc. of ECOC 2016, paper W.1.D.3, Amsterdam (The Netherlands), Sep. 2016.

P. Poggiolini et al., "Non-Linearity Modeling at Ultra-High Symbol Rates," Proc. Of OFC 2018, San Diego (USA), Mar. 2018.

32 Gbaud - 48 channels - SMF - 100km spans



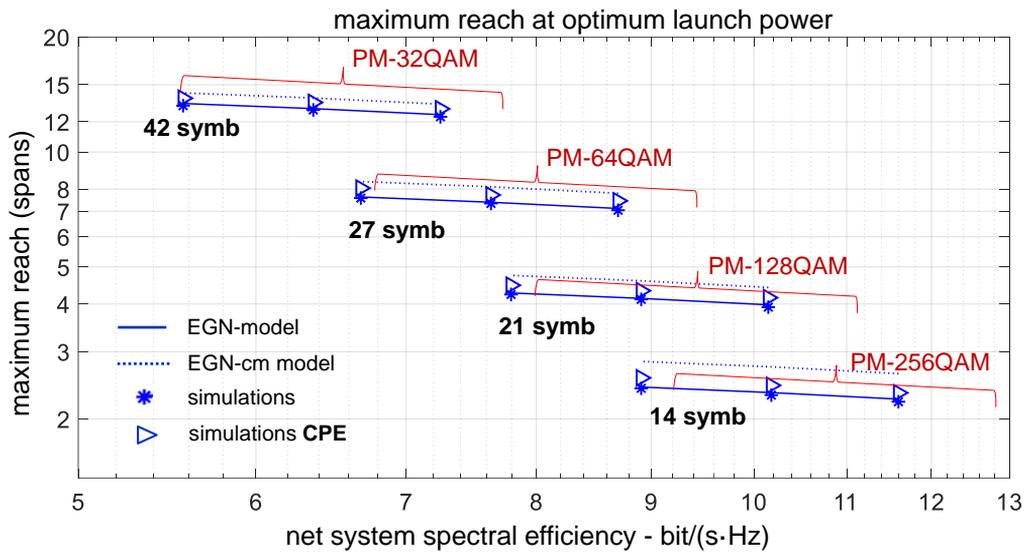
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32 Gbaud - 48 channels - SMF - 100km spans



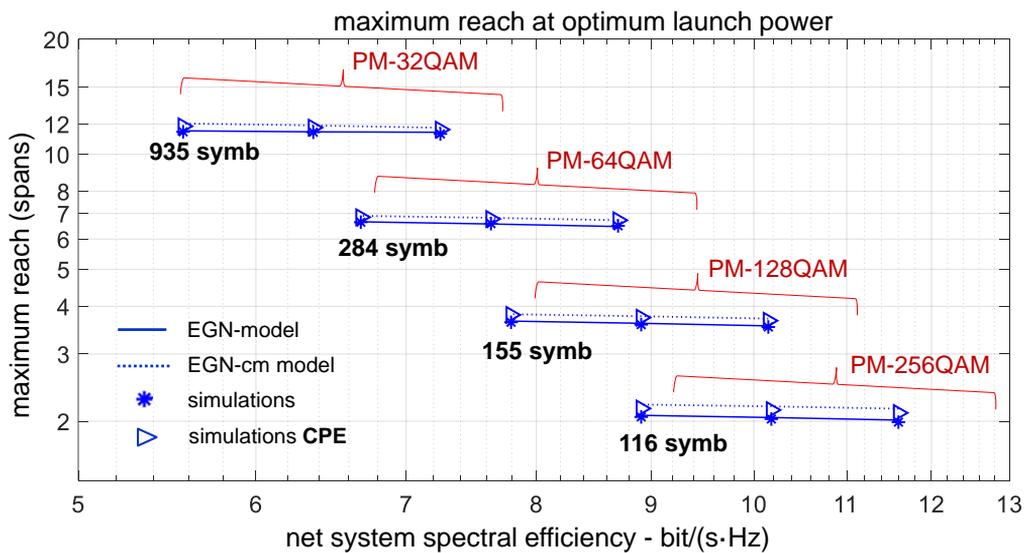
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256 Gbaud - 6 channels - SMF - 100km spans



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Going towards high symbol rates ...



- NLPN decreases, as shown by the EGN-cm model
- Mitigating it is easier

- Overall, its impact decreases

P. Poggiolini et al., "Non-Linearity Modeling at Ultra-High Symbol Rates," Proc. Of OFC 2018, San Diego (USA), Mar. 2018.

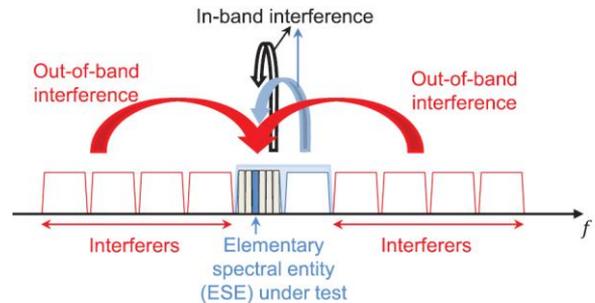


MITIGATION OF FIBER NONLINEARITY

NLI mitigation techniques

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- Several nonlinearity mitigation techniques have been proposed to reduce the power of the NLI noise.
- The effectiveness of nonlinearity mitigation depends on the nature of the NLI, which can be divided into two main classes:
 - **In-band interference**, which includes the NLI generated within the electronic bandwidth of the transceiver
 - **Out-of-band interference**, which includes the NLI generated by the interaction with WDM channels that are not accessible to transmitter and receiver.



NLI mitigation techniques

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- **In-band NLI** can be mitigated by means of:
 - digital backpropagation (DBP)
 - nonlinear Fourier transform (NFT) transmission techniques
- **Out-of-band NLI that involves the channel under test** can be modelled as time-varying ISI and can be partially mitigated using several techniques (such as MAP or ML decoding).
- **Out-of-band NLI generated solely by out-of-band channels** is typically treated as non-removable noise.
- Mitigation of the impact of NLI can be also obtained using different approaches that optimize one or more transmission parameters, among which:
 - symbol-rate optimization (SRO)
 - dispersion pre-compensation
 - constellation or pulse shaping



Ideal gains of SRO and DBP

The analyzed set-up

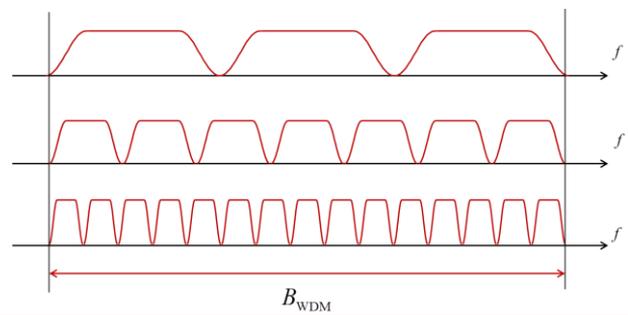


- **What is the symbol rate which minimizes NLI ?...**

...having fixed:

- the total WDM bandwidth ($B_{\text{WDM}}=504 \text{ GHz}, 1.5 \text{ THz}, 2.5 \text{ THz}, 5 \text{ THz}$)
- the modulation format and roll-off (PM-QPSK or PM-16QAM, $\rho=0.05$)
- the relative frequency spacing ($\Delta f=1.05 R_s$)

- EDFA-only amplification ($F=5 \text{ dB}$)
- SSMF fiber (100-km span length)



Normalized NLI power spectral density

- The total NLI power (P_{NLI}) at the output of the transmission link is estimated either with the EGN model or by numerical simulations based on the split-step Fourier method.
- Systems at different symbol rate are compared in terms of the **normalized NLI power spectral density (PSD)**

$$\tilde{G}_{NLI} = \frac{P_{NLI}}{R_s G_{ch}^3}$$

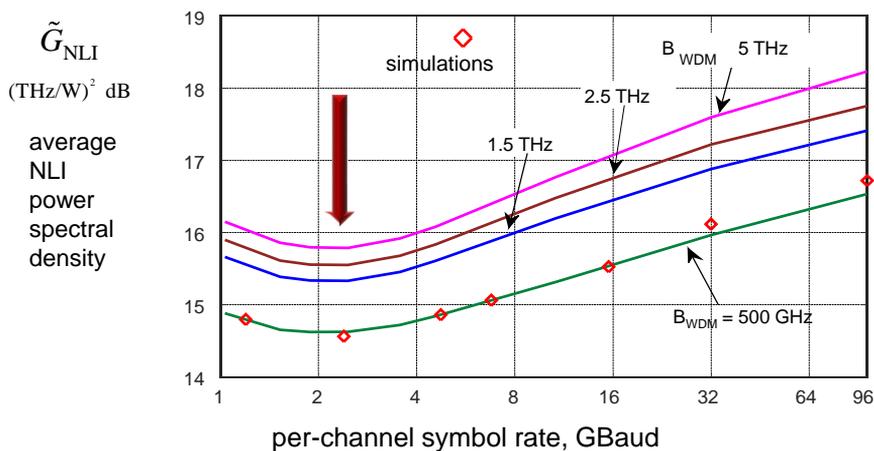
Transmitted signal PSD

which is independent of the transmitted power per channel.

- Same value of \tilde{G}_{NLI} means same maximum reach.

SRO prediction by EGN model

- ▶ **PM-QPSK**, roll-off 0.05, spacing 1.05 x (symb rate), **SMF**, 100 km spans, 50 spans



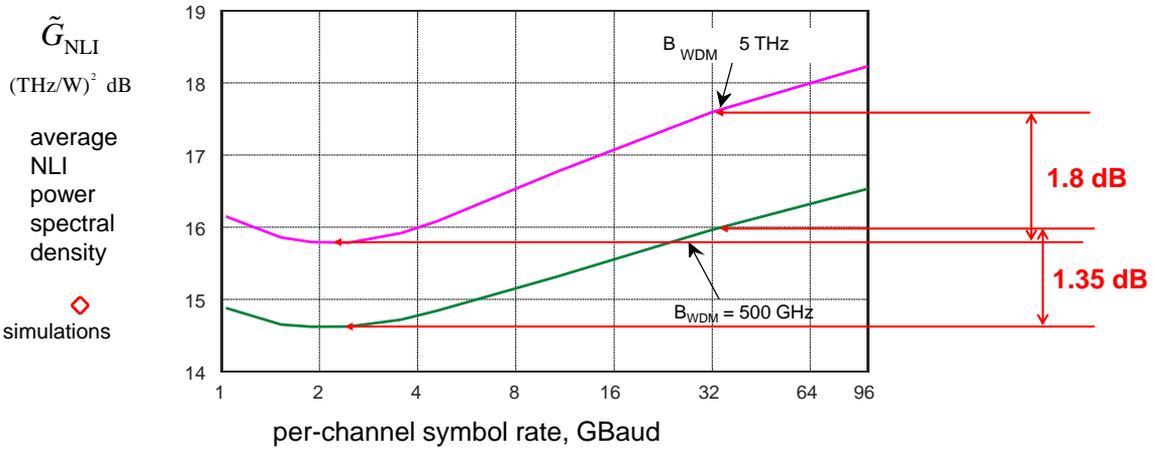
All curves drawn using the **FULL EGN model**

$$R_{opt} = \sqrt{\frac{2}{\pi |\beta_2| N_{span} L_{span}}}$$

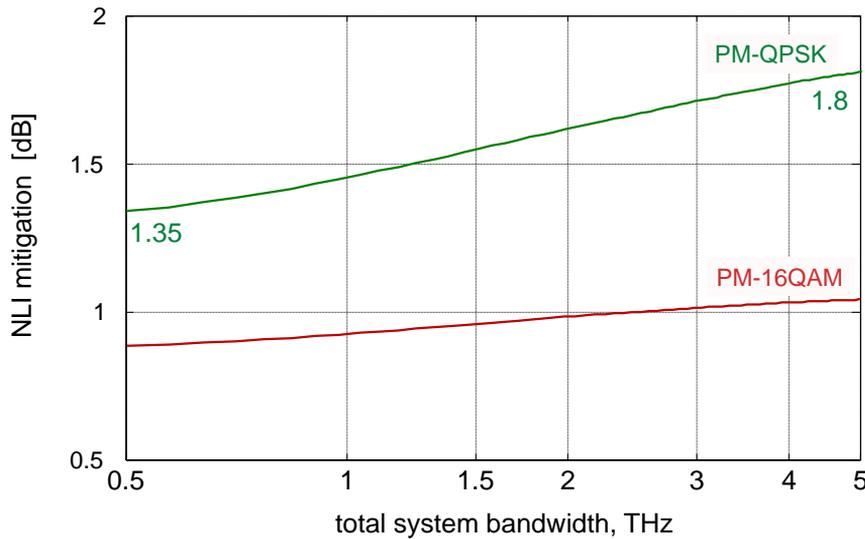
SRO prediction by EGN model



► **PM-QPSK, roll-off 0.05, spacing 1.05 x (symb rate), SMF, 100 km spans, 50 spans**



NLI mitigation



All curves drawn using the FULL EGN model

SRO between 32 GBaud and the optimum rate (2.4 GBaud)

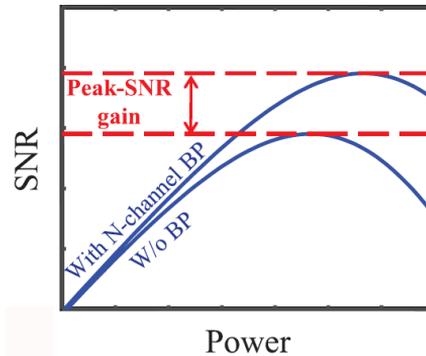
Max reach and peak-SNR

- How does **NLI mitigation** translate into **max reach or peak-SNR gains**?

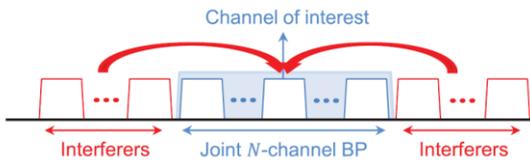
$$\Delta SNR_{peak,dB} = -\frac{1}{3} \Delta \tilde{G}_{NLI,dB}$$

$$\Delta L_{max,dB} \approx -\frac{1}{3} \Delta \tilde{G}_{NLI,dB}$$

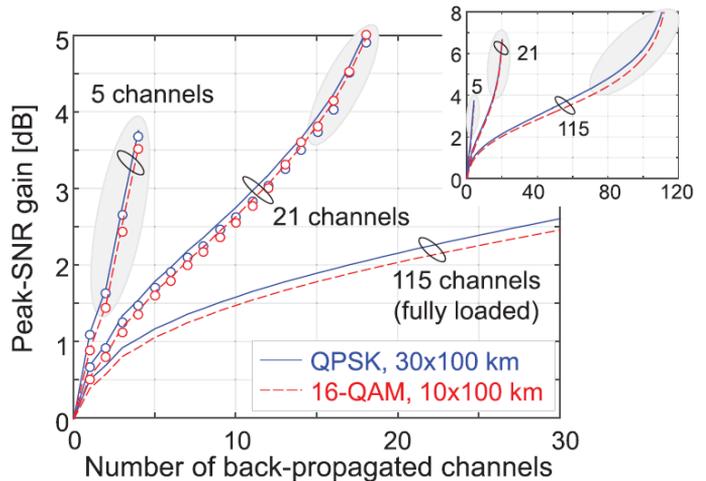
- 1.8 dB mitigation → 15% max reach gain
- 1.35 dB mitigation → 11% max reach gain



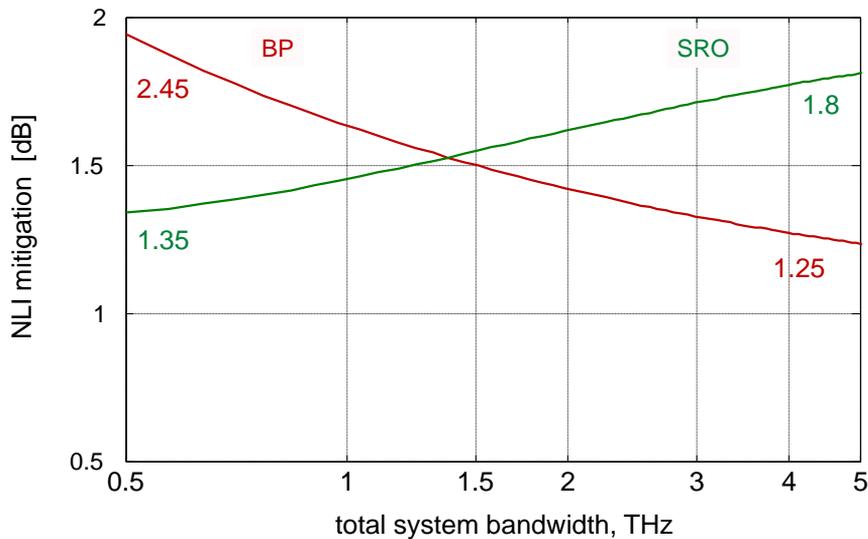
Prediction of DBP gain



R. Dar and P. Winzer, "Nonlinear Interference Mitigation: Methods and Potential Gain," *J. Lightw. Technol.* **35**(4), p. 903 (2017).



Backward Propagation vs. SRO – PM-QPSK

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PM-QPSK, roll-off 0.05, spacing 1.05 x (symb rate), **SMF, 100 km spans, 50 spans**

All curves drawn using the FULL EGN model

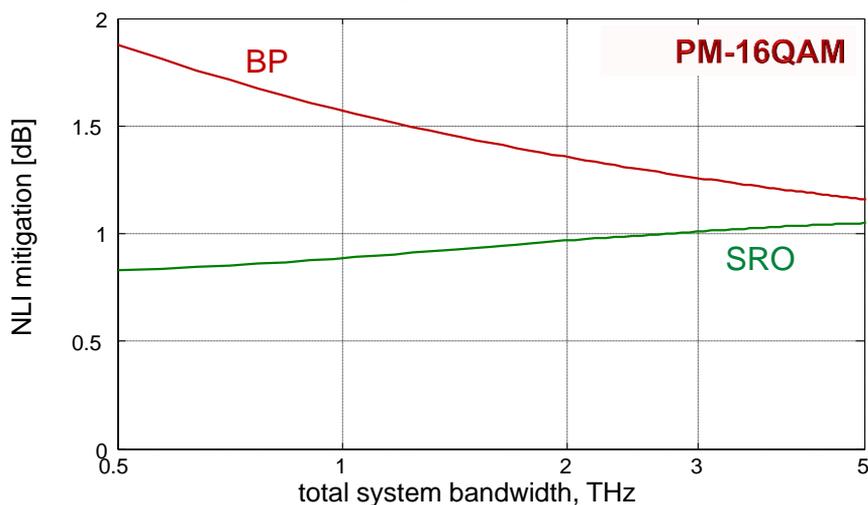
SRO: between 32 GBaud and the optimum rate (2.4 GBaud)

BP: *ideal* backward propagation over the 32 GBaud center channel

Backward Propagation vs. SRO – PM-16QAM

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► **PM-16QAM**, roll-off 0.05, spacing 1.05 x (symb rate), **SMF, 100 km spans, 50 spans**



SRO: between 32 GBaud and the optimum rate (2.4 GBaud)

BP: *ideal* backward propagation over the 32 GBaud center channel



Practical limitations of SRO and DBP

How to exploit symbol-rate optimization gain?



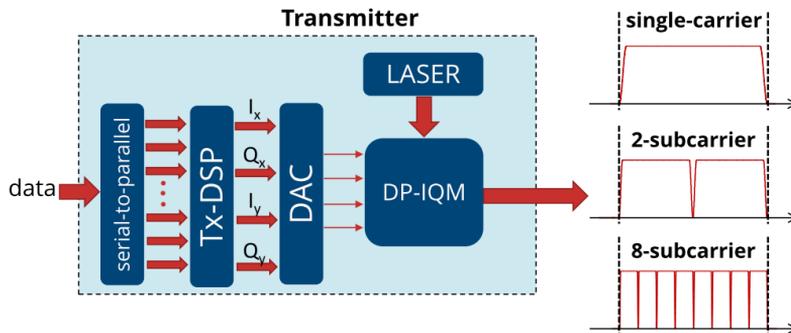
- Optimum symbol rate values in the range 2-4 Gbaud
- It would be extremely inefficient to use a separate transceiver for each low-symbol-rate signal
 - To reach the transmission speed of commercially available 32-Gbaud systems, 16x more transceivers (including laser sources) at 2 Gbaud would be required



Sub-carrier multiplexing

- A high symbol-rate signal is electrically decomposed into a given number of subcarriers, each of which operating at a lower symbol-rate (multiplexing in the digital domain)

Subcarrier multiplexing

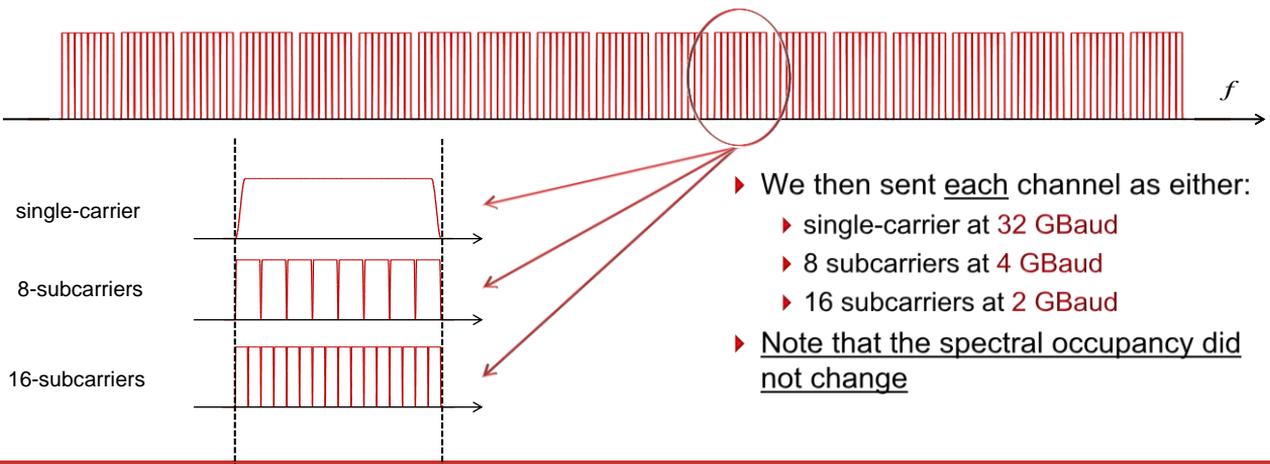
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- A. Nespola et al., "Experimental demonstration of fiber nonlinearity mitigation in a WDM multi-subcarrier coherent optical system," ECOC 2015, Sep. 2015.

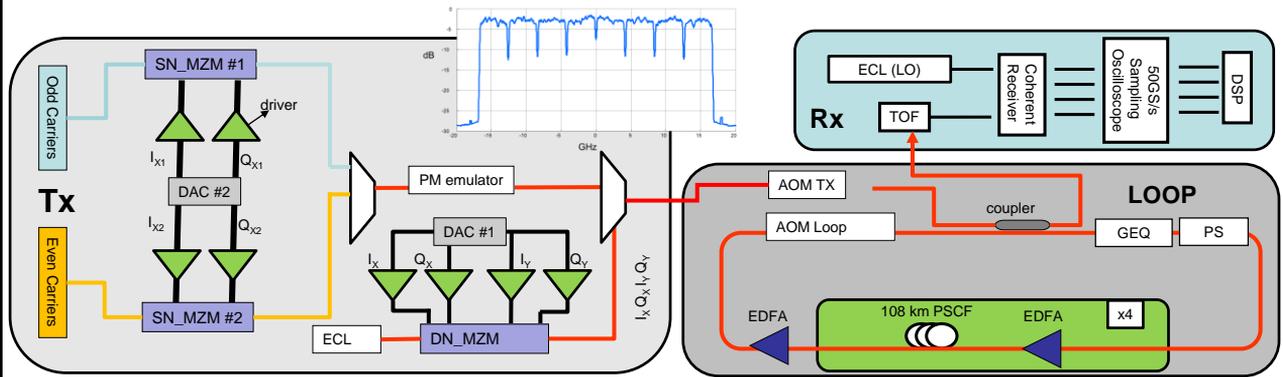
The experiment

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- We started out with a **19 channel WDM comb**, with channel **spacing 37.5 GHz**, for a total WDM bandwidth of **710 GHz**



System schematic



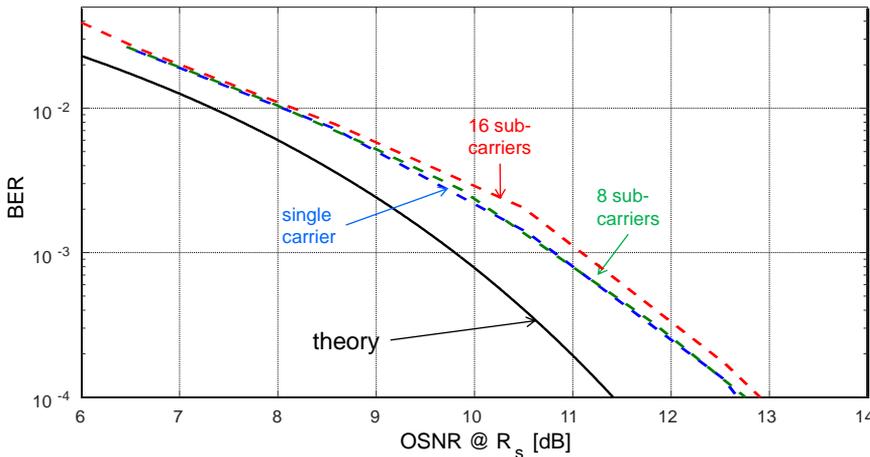
SN_MZM: single-nested Mach-Zehnder mod.
 DN_MZM: double-nested Mach-Zehnder mod.

GEQ: Gain Equalizing programmable filter
 PS: synchronous Polarization Scrambler
 AOM: Acousto-Optic Modulator (used as switch)
 TOF: Tunable Optical Filter

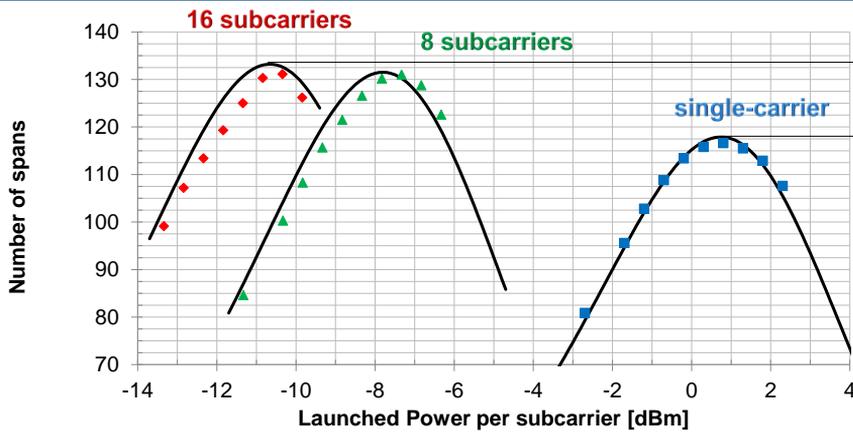
Back-to-back characterization



To perform a meaningful comparative test over the long-haul, it is important that the btb is the same



- At the reference BER=10⁻²:
 - No penalty from single SC to 8 SCs
 - 0.1-dB penalty from single SC to 16 SCs

Reach curves at BER 10^{-2} POLITECNICO
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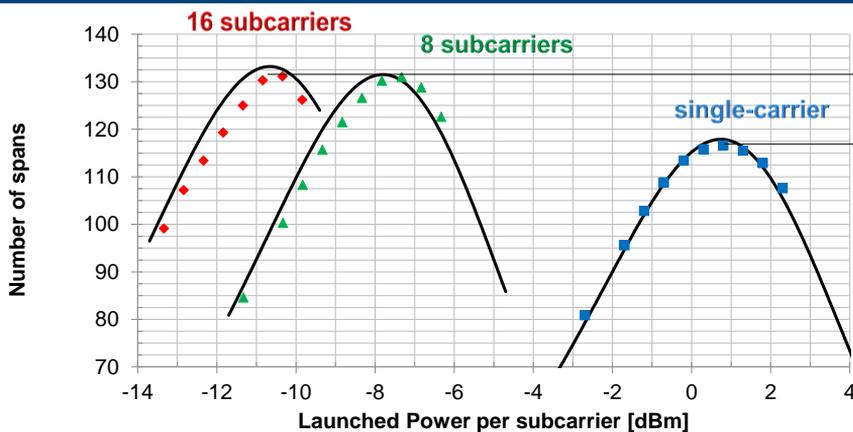
13.5 % reach increase

- ▶ markers: experiment
- ▶ solid lines: EGN model predictions

- Back-to-back penalty taken into account in the EGN model curves (theoretical reach gain $\sim 14.5\%$)

OPTCOM

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Reach curves at BER 10^{-2} POLITECNICO
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12.5 % reach increase

- ▶ markers: experiment
- ▶ solid lines: EGN model predictions

- The gain predicted by the analytical model cannot be fully exploited due to practical implementation issues (higher sensitivity to transceiver impairments and phase noise)

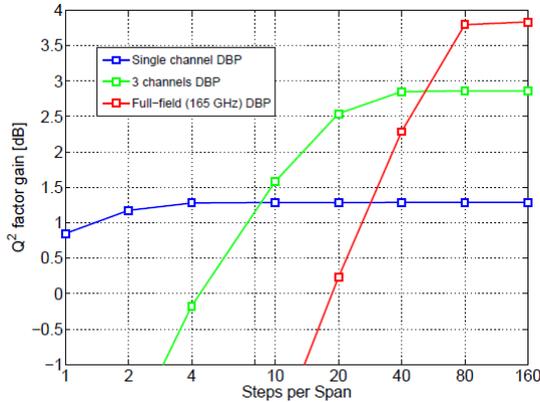
OPTCOM

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DBP performance vs. number of steps per span

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- ▶ 5x32 Gbaud PM-16QAM, SMF, 80 km spans, 40 spans



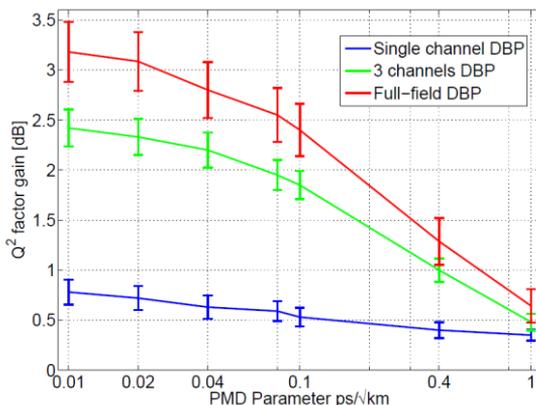
- The number of steps per span must be increased as the DBP bandwidth increases.
- The benefit of back-propagating large bandwidths can be totally lost if a suboptimal step size is used (due to an inaccurate compensation of the NLI).

- G. Liga et al., "On the performance of multichannel digital backpropagation in high-capacity long-haul optical transmission," *Opt. Exp.* **22**(24), p. 30053 (2014)

DBP performance vs. PMD

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- ▶ 5x32 Gbaud PM-16QAM, SMF, 80 km spans, 40 spans



- Multichannel DBP performed with optimal operation parameters.
- For typical values of the PMD parameter the beneficial effects of multichannel DBP reduce significantly as the DBP bandwidth is increased.
- In the presence of PMD, back-propagating larger portions of bandwidth beyond a certain value becomes ineffective to improve the transmission performance.

- G. Liga et al., "On the performance of multichannel digital backpropagation in high-capacity long-haul optical transmission," *Opt. Exp.* **22**(24), p. 30053 (2014)

Conclusions



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- The **NLI analytical models** are useful tools to obtain an accurate prediction of the **ultimate performance** achievable by the various mitigation techniques.
- The **actual performance gain** will also depend on several implementation issues that cannot be easily included in the analytical estimations, such as:
 - sub-optimum performance of low-complexity DBP algorithms
 - higher impact of NLPN in digital multi-subcarrier systems
 which reduce the nonlinearity mitigation benefits.

Symposium on Challenges to Achieving Capacity in Nonlinear Optical Networks



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Modelling and mitigation of fibre nonlinearity in WDM coherent optical systems

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