A Novel Update Algorithm in Stokes Space for Adaptive Equalization in Coherent Receivers

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Coherent detection and DSP

- High-order modulation
- Coherent detection
- Digital signal processing

High spectral efficiency transmission over long-haul optical links
Coherent detection and DSP

- Front-end IQ imbalance
- Chromatic dispersion
- PMD
- PDL
- Residual CD
- Filtering effects
- Frequency offset
- Laser phase noise
- Fiber non-linear effects

High-order modulation

Coherent detection

Digital signal processing

High spectral efficiency transmission over long-haul optical links

Adaptive MIMO equalizer
MIMO equalizer

- Standard CMA or LMS algorithms: update of the coefficients based on error signals evaluated on the two-dimensional constellations (separate for the two polarizations)

- New algorithm: update of the coefficients based on error signal evaluated in the Stokes space

  - Performance test on a PM-16QAM signal, comparing it to the multi-modulus CMA algorithm
PM-QPSK

\[ S_1(n) = |E_x(n)|^2 - |E_y(n)|^2 \]
\[ S_2(n) = 2\Re\{E_x(n)E_y(n)\} \]
\[ S_2(n) = 2\Im\{E_x(n)E^*_y(n)\} \]

JONES SPACE

STOKES SPACE
\[ S_1(n) = |E_x(n)|^2 - |E_y(n)|^2 \]
\[ S_2(n) = 2 \Re \{ E_x(n) E_y(n) \} \]
\[ S_2(n) = 2 \Im \{ E_x(n) E^*(n) \} \]
Design rule Stokes equalizer

- Error function to be minimized

\[ f(h) = f(h_{xx}, h_{xy}, h_{yx}, h_{yy}) = \]
\[ = (S_{1e}(n) - \hat{S}_1(n))^2 + (S_{2e}(n) - \hat{S}_2(n))^2 + (S_{3e}(n) - \hat{S}_3(n))^2 \]

with:

- \( S_e = [S_{1e}(n), S_{2e}(n), S_{3e}(n)] \)  
  Stokes vector of the equalized signal

- \( \hat{S} = [\hat{S}_1(n), \hat{S}_2(n), \hat{S}_3(n)] \)  
  Stokes vector of the transmitted signal

either known (training sequence) or estimated (decision-directed)
Rule for adaptively update the equalizer weights:
\[
\begin{align*}
    h_{xx}(n+1) &= h_{xx}(n) - \mu \nabla_{h_{xx}} f(h(n)) \\
    h_{xy}(n+1) &= h_{xy}(n) - \mu \nabla_{h_{xy}} f(h(n)) \\
    h_{yx}(n+1) &= h_{yx}(n) - \mu \nabla_{h_{yx}} f(h(n)) \\
    h_{yy}(n+1) &= h_{yy}(n) - \mu \nabla_{h_{yy}} f(h(n))
\end{align*}
\]

Evaluation of gradients:
\[
\begin{align*}
    \nabla_{h_{xx}} f(h(n)) &= C_1(n) E_x^* \\
    \nabla_{h_{xy}} f(h(n)) &= C_1(n) E_y^* \\
    \nabla_{h_{yx}} f(h(n)) &= C_2(n) E_y^* \\
    \nabla_{h_{yy}} f(h(n)) &= C_2(n) E_x^*
\end{align*}
\]

Stokes algorithm
\[
\begin{bmatrix}
    C_1(n) \\
    C_2(n)
\end{bmatrix} =
\begin{bmatrix}
    \varepsilon_1(n) & \varepsilon_2(n) \\
    \varepsilon^*_2(n) & -\varepsilon_1(n)
\end{bmatrix}
\begin{bmatrix}
    E_{xe}(n) \\
    E_{ye}(n)
\end{bmatrix}
\]
\[
\begin{align*}
    \varepsilon_1(n) &= S_{1e}(n) - \hat{S}_1(n) \\
    \varepsilon_2(n) &= (S_{2e}(n) - \hat{S}_2(n)) + j(S_{3e}(n) - \hat{S}_3(n))
\end{align*}
\]

CMA
\[
\begin{bmatrix}
    C_1(n) \\
    C_2(n)
\end{bmatrix} =
\begin{bmatrix}
    \varepsilon_x(n) & 0 \\
    0 & \varepsilon_y(n)
\end{bmatrix}
\begin{bmatrix}
    E_{xe}(n) \\
    E_{ye}(n)
\end{bmatrix}
\]
\[
\begin{align*}
    \varepsilon_x(n) &= |E_{xe}(n)|^2 - R^2 \\
    \varepsilon_y(n) &= |E_{ye}(n)|^2 - R^2
\end{align*}
\]
Case study – PM-16QAM

- Symbol rate: $R_s = 32$ Gbaud
- Single-channel
- Nyquist spectrum (raised-cosine with roll-off of 0.1)
- Residual CD = 250 ps/nm
- DGD = 1 symbol

- BER values estimated through Monte-Carlo simulation for several combinations of DGD axis and state of polarization (SOP) at the input of the Rx, for a total of ~900 cases

- Equalization using a training sequence, followed by decision-directed operation
The value of the adaptive equalizer update coefficient $\mu$ was optimized for both CMA and Stokes algorithms.
Performance can be improved by:

1. Using an adaptive Maximum-Likelihood decision criterion instead of a fixed-threshold one

2. Changing the decision rule in the Stokes space (minimum distance is not optimum)
Statistics of noise in Stokes space

- \( \mathbf{S} = (S_1, S_2, S_3) \) = noisy received vector
- \( \mathbf{\hat{S}}_i = (\hat{S}_{i1}, \hat{S}_{i2}, \hat{S}_{i3}) \) = ideal un-noisy constellation vector

PDF of \( \mathbf{S} \mid \mathbf{\hat{S}}_i \) [*]:

\[
f_{\mathbf{S} \mid \mathbf{\hat{S}}_i} = e^{-\frac{\hat{S}_{0i} + S_0}{2\sigma^2}} I_0 \left( \frac{\sqrt{\hat{S}_{0i} \cdot S_0}}{\sigma^2} \cos \left( \frac{\theta_i}{2} \right) \right)
\]

- \( S_0 \) = magnitude of \( \mathbf{S} \)
- \( \hat{S}_{0i} \) = magnitude of \( \mathbf{\hat{S}}_i \)
- \( \theta_i \) = angle between \( \mathbf{S} \) and \( \mathbf{\hat{S}}_i \)
- \( \sigma^2 \) = noise variance in each polarization

The decision rule can be based on the maximization of $f_{S|\hat{S}_i}$ over all possible noiseless constellation points $\hat{S}_i$.

There are common factors across all possible indices $i$ that can be eliminated $\rightarrow$ we can apply the ML decision on the formula:

$$p_i = e^{-\frac{s_0}{2\sigma^2}} I_0 \left( \frac{\sqrt{\hat{S}_{0_i} \cdot S_0}}{\sigma^2} \cos \left( \frac{\theta_i}{2} \right) \right)$$
Taking the logarithm and applying some simplifications, we obtain the following new metric (based on actual statistics in Stokes space):

\[ m_i \approx -S_{0_i} + 2 \sqrt{\hat{S}_{0_i}} \sqrt{S_0} \cos \left( \frac{\theta_i}{2} \right) \]

Minimum-distance metric (based on Gaussian distribution hypothesis)

\[ d_i^2 = (S_1 - \hat{S}_{1_i})^2 + (S_2 - \hat{S}_{2_i})^2 + (S_3 - \hat{S}_{3_i})^2 = S_0^2 + \hat{S}_{0_i}^2 - 2 \hat{S}_{0_i} S_0 \cos(\theta_i) \]

\[ \iff \quad -\hat{S}_{0_i}^2 + 2\hat{S}_{0_i} S_0 \cos(\theta_i) \]
16QAM with adaptive ML receiver

- Number of eq. taps: $M = 31$.
- Solid lines: average performance (over all considered values of SOP and DGD axis).

- The value of the adaptive equalizer update coefficient $\mu$ was optimized for both CMA and Stokes algorithms.
Phase noise tolerance

- CPE block (inserted after the MIMO equalizer), based on the Viterbi&Viterbi algorithm, with QPSK partitioning
- Stokes-space update guarantees that the two polarizations after equalization are perfectly aligned to each other in phase → the phase error estimate can be obtained as an average on both polarizations

- 32 Gbaud PM-16QAM
- Reference BER: $2 \cdot 10^{-2}$
A novel update algorithm for adaptive MIMO equalizer taps has been proposed and its performance analyzed in a PM-16QAM scenario.

At the expenses of a slight increase in complexity, it has the following advantages with respect to standard CMA and LMS algorithms:

**With respect to LMS**
- Insensitive to phase noise and frequency offset

**With respect to CMA**
- Not affected by the “degeneracy” problem typical of CMA
- CPE algorithms can estimate the phase noise by averaging over the two polarizations \( \rightarrow \) nearly doubled phase-noise tolerance
Thank you!

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