Analytical Results on Channel Capacity in Uncompensated Optical Links with Coherent Detection

Gabriella Bosco, Pierluigi Poggiolini, Andrea Carena, Vittorio Curri
Politecnico di Torino

Fabrizio Forghieri
Cisco Photonics Italy

ECOC 2011 - paper We.7.B.3
Outline

- Theory
  - Shannon formulas
  - Capacity of the polarization-multiplexed (PM) optical channel

- Results for uncompensated systems with EDFA amplification
  - Gaussian constellation
  - Realistic constellations with hard and soft decoding
Shannon formulas

- Capacity of the unconstrained AWGN channel:
  \[ C = \log_2(1 + \text{SNR}) \text{ [bits/symbo l]} \]

- Capacity of the polarization-multiplexed (PM) optical channel:
  \[
  C = 2 \frac{R_s}{\Delta f} \log_2(1 + \text{SNR}) \text{ [bits/symbo l]}
  \]

  with

  \[ \text{SNR} = \frac{B_n}{R_s} \text{OSNR} \]

- \( R_s = \text{symbol-rate} \)
- \( \Delta f = \text{frequency spacing between WDM channels} \)
- \( B_n = \text{reference noise bandwidth} \)
According to the models presented in [1],[2], the system BER depends on a “generalized” OSNR:

\[
\text{OSNR}_{\text{NL}} = \frac{P_{\text{Tx, ch}}}{P_{\text{ASE}} + P_{\text{NLI}}}
\]

In case of EDFA amplification:

\[
P_{\text{ASE}} = N_s F \left( e^{2\alpha L_s} - 1 \right) h \nu B_n
\]

- \(P_{\text{Tx, ch}}\) = signal power
- \(P_{\text{ASE}}\) = ASE noise power
- \(P_{\text{NLI}}\) = non-linear interference (NLI) power
- \(N_s\) = number of fiber spans
- \(F\) = EDFA noise figure
- \(\alpha\) = fiber loss coefficient
- \(L_s\) = length of fiber span
- \(h\) = Plank’s constant
- \(\nu\) = center frequency


NLI power at Nyquist limit

- At the Nyquist limit

the power of the non-linear interference can be analytically evaluated in uncompensated optical systems [3]:

\[ P_{\text{NLI}} \approx \left( \frac{2}{3} \right)^3 N_s \gamma^2 L_{\text{eff}}^3 G_{\text{Tx}}^3 \frac{\ln(\pi^2 |\beta_2| L_{\text{eff}} B_{\text{WDM}}^2)}{\pi |\beta_2|} B_n \]

\[ G_{\text{Tx}} = \frac{P_{\text{Tx, ch}}}{R_s} \quad B_{\text{WDM}} = N_{\text{ch}} R_s \]

\[ L_{\text{eff}} = \frac{1 - e^{-2\alpha L_s}}{2\alpha} \]

- \( \beta_2 \) = dispersion coefficient
- \( \gamma \) = non-linearity coeff.
- \( L_{\text{eff}} \) = fiber effective length

Capacity of a PM optical channel

\[ C = 2 \log_2 \left( 1 + G_{Tx} \left[ N_s \left( e^{2\alpha L_s} - 1 \right) F h \nu + \left( \frac{2}{3} \right)^3 \gamma^2 N_s L_{eff} G_{Tx}^3 \frac{\ln \left( \frac{\pi^2 |\beta_2| L_{eff} B_{WDM}^2}{\pi |\beta_2|} \right)}{\pi |\beta_2|} \right] \right) \]

\[ G_{Tx} = \frac{P_{Tx,ch}}{R_s} \quad B_{WDM} = N_{ch} R_s \]

- \( N_s \) = number of spans
- \( L_s \) = length of fiber span
- \( \beta_2 \) = dispersion coefficient
- \( \gamma \) = non-linearity coeff.
- \( \alpha \) = fiber loss coefficient
- \( L_{eff} \) = fiber effective length
- \( F \) = EDFA noise figure
- \( h \) = Plank’s constant
- \( \nu \) = center frequency
- \( R_s \) = symbol-rate
Capacity of a PM optical channel

\[
C = 2 \log_2 \left( 1 + G_{Tx} \left[ N_s \left(e^{2\alpha L_s} - 1\right) F h \nu + \left(\frac{2}{3}\right)^3 \gamma^2 N_s L_{eff} G_{Tx}^3 \frac{\ln\left(\pi^2 |\beta_2| L_{eff} B_{WDM}^2\right)}{\pi |\beta_2|} \right] \right) \right)
\]

\[
G_{Tx} = \frac{P_{Tx, ch}}{R_s} \quad B_{WDM} = N_{ch} R_s
\]

- \(N_s\) = number of spans
- \(L_s\) = length of fiber span
- \(\beta_2\) = dispersion coefficient
- \(\gamma\) = non-linearity coeff.
- \(\alpha\) = fiber loss coefficient
- \(L_{eff}\) = fiber effective length
- \(F\) = EDFA noise figure
- \(h\) = Plank’s constant
- \(\nu\) = center frequency
- \(R_s\) = symbol-rate

**Hypotheses:**
- EDFA amplification
- Uncompensated transmission
- Nyquist limit (\(\Delta f = R_s\))
- Ideal Gaussian constellation
- Soft decision
Capacity of a PM optical channel

\[ C = 2 \log_2 \left( 1 + G_{Tx} \left[ N_s \left( e^{2\alpha L_s} - 1 \right) F h \nu + \left( \frac{2}{3} \right)^3 \gamma^2 N_s L_{eff} G_{Tx}^3 \ln \left( \frac{\pi^2 |\beta_2| L_{eff} B_{WDM}^2}{\pi |\beta_2|} \right) \right] \right) \]

\[ G_{Tx} = \frac{P_{Tx,ch}}{R_s} \]

\[ B_{WDM} = N_{ch} R_s \]

- \( N_s = \) number of spans
- \( L_s = \) length of fiber span
- \( \beta_2 = \) dispersion coefficient
- \( \gamma = \) non-linearity coeff.
- \( \alpha = \) fiber loss coefficient
- \( L_{eff} = \) fiber effective length
- \( F = \) EDFA noise figure
- \( h = \) Plank’s constant
- \( \nu = \) center frequency
- \( R_s = \) symbol-rate

At the Nyquist limit, capacity is independent of the symbol-rate.
$B_{WDM} = 4$ THz (C-band)

SSMF fiber
- $L_s = 100$ km
- $\gamma = 1.27$ W$^{-1}$km$^{-1}$
- $\beta_2 = -21.7$ ps$^2$/km
- $\alpha_{dB} = 0.22$ dB/km
- $F = 5$ dB
- $\nu = 193$ THz

The optimum PSD does not depend on transmission distance
One relevant feature of previous figure is that the optimum launch power (or signal PSD) is the same for every distance.
One relevant feature of previous figure is that the optimum launch power (or signal PSD) is the same for every distance

\[ G_{Tx, opt} = \frac{3}{2^\frac{4}{3}} \left( \frac{\left( e^{2\alpha L_s} - 1 \right) F h v \pi |\beta_2|}{\gamma^2 L_{eff} \log \left( \pi^2 |\beta_2| L_{eff} B_{WDM}^2 \right)} \right)^{\frac{1}{3}} \]
One relevant feature of previous figure is that the optimum launch power (or signal PSD) is the same for every distance:

\[
G_{Tx,\text{opt}} = \frac{3}{2^\frac{4}{3}} \left( \frac{\left(e^{2\alpha s} - 1\right)Fh \upsilon \pi |\beta_2|}{\gamma^2 L_{\text{eff}} \log\left(\pi^2 |\beta_2| L_{\text{eff}} B_{\text{WDM}}^2\right)} \right)^{\frac{1}{3}}
\]

For fixed amplifier noise figure and total bandwidth occupancy, the optimum launch power is independent of the number of spans (and consequently of the total link length).

It indeed depends on fiber parameters (span length, fiber loss, dispersion, nonlinearity coefficient and effective length).
To obtain capacity estimates for generic PM coherent formats in UT links, the standard formulas of capacity over AWGN [4], specific of each format, should be used, with the SNR derived from the generalized OSNR expression:

\[ \text{SNR} = \frac{B_n}{R_s} \text{OSNR}_{NL} \]

\[
\text{OSNR}_{NL} = \frac{R_s}{B_n} \ln \left( \frac{e^{2\alpha L_s} - 1}{Fh\nu} + \left( \frac{2}{3} \right)^3 \gamma^2 N_s L_{\text{eff}} G_{Tx,ch}^3 \right) + \ln \frac{\pi^2 |\beta_2| L_{\text{eff}} B_{WDM}^2}{\pi |\beta_2|} 
\]

Hard and soft-decision capacity

**HARD DECISION**

\[
C = 2 \frac{1}{M} \sum_{a,b} P_{Y|X}(b|a) \log_2 \frac{P_{Y|X}(b|a)}{P_Y(b)}
\]

**SOFT DECISION**

\[
C = 2 \frac{1}{M} \sum_{a} \int P_{Y|X}(y|a) \log_2 \frac{P_{Y|X}(y|a)}{P_Y(y)}
\]

- All symbols are assumed to have the same transmission probability
- \(X=\{x_1,..,x_M\}\) is the input alphabet
- \(Y=\{y_1,..,y_M\}\) is the hard-decision output alphabet
- \(y\) is the soft value at the output of the channel
- \(P_{Y|X}(b|a)\)=probability of receiving \(b\) when \(a\) has been transmitted
- \(P_Y(b)\)= probability of receiving each of the constellation symbols

In an AWGN channel:

\[
p_{Y|X}(y|a) = \frac{1}{\pi \sigma_N^2} e^{-\frac{d^2(a,y)}{\sigma_N^2}}
\]
EDFA amplification (1000 km)

- SSMF fiber
- $L_s = 100$ km
- $N_s = 10$
- $\Delta f = R_s$
- $B_{WDM} = 4$ THz
- $\nu = 193$ THz
- $F = 5$ dB

Soft decision

<table>
<thead>
<tr>
<th>Modulation</th>
<th>BER at maximum</th>
</tr>
</thead>
<tbody>
<tr>
<td>PM-QPSK</td>
<td>$6.4 \cdot 10^{-8}$</td>
</tr>
<tr>
<td>PM-16QAM</td>
<td>$6.8 \cdot 10^{-3}$</td>
</tr>
<tr>
<td>PM-64QAM</td>
<td>$7.2 \cdot 10^{-2}$</td>
</tr>
</tbody>
</table>
EDFA amplification (1000 km)

- SSMF fiber
- $L_s = 100$ km
- $N_s = 10$
- $\Delta f = R_s$
- $B_{WDM} = 4$ THz
- $\nu = 193$ THz
- $F = 5$ dB

The optimum PSD does not depend on modulation format:

<table>
<thead>
<tr>
<th>Modulation Format</th>
<th>BER at maximum</th>
</tr>
</thead>
<tbody>
<tr>
<td>PM-QPSK</td>
<td>$6.4 \times 10^{-8}$</td>
</tr>
<tr>
<td>PM-16QAM</td>
<td>$6.8 \times 10^{-3}$</td>
</tr>
<tr>
<td>PM-64QAM</td>
<td>$7.2 \times 10^{-2}$</td>
</tr>
</tbody>
</table>

Solid lines: soft decision
Dashed lines: hard decision

26.8 $\mu$W/GHz
EDFA amplification (5000 km)

Solid lines: soft decision
Dashed lines: hard decision

- SSMF fiber
- $L_s = 100$ km
- $N_s = 50$
- $\Delta f = R_s$
- $B_{WDM} = 4$ THz
- $\nu = 193$ THz
- $F = 5$ dB

The optimum PSD does not depend on transmission distance

26.8 $\mu$W/GHz
SSMF with 100 km span length

- Maximum capacity (at optimum PSD) vs. distance

![Graph showing capacity vs. distance for different modulation formats (PM-Gaussian, PM-64QAM, PM-16QAM, PM-QPSK).](image)

- Fixing the distance, the capacity penalty of each format with respect to its maximum corresponds to the required FEC overhead.

- 20% hard-FEC overhead

**Trade-off between capacity and distance**

[f] = Rs
PSCF with 50 km span length

- Maximum capacity (at optimum PSD) vs. distance

- Solid lines: soft decision
- Dashed lines: hard decision

- **PSCF fiber**
  - $\gamma = 1.0$ W$^{-1}$km$^{-1}$
  - $\beta_2 = -26.2$ ps$^2$/km
  - $\alpha_{\text{dB}} = 0.18$ dB/km

20% hard-FEC overhead

- **Capacity (bits/symbol)**
  - PM-Gaussian constellation
  - PM-64QAM
  - PM-16QAM
  - PM-QPSK

- **Distance (km)**
  - 2500 km
  - 8000 km

www.optcom.polito.it
Conclusions

- Non-linear propagation model for uncompensated transmission (validated both through simulations and experiments)

- Analytical form of the channel capacity at the Nyquist limit.

- Capacity is independent of the symbol-rate.
  Optimum launch power spectral density is independent of link length.

- Examples of application to uncompensated optical systems with EDFA amplification.

- The obtained results can be extended to the general case of non-rectangular spectra and spacing larger than the symbol-rate.
This work was partially supported by CISCO Systems within a SRA contract.

This work was supported by the European Union within the EURO-FOS project, a Network of Excellence funded by the European Commission through the 7th ICT-Framework Programme.