

# System Impact of Parametric Gain: a Novel Method for the BER Evaluation

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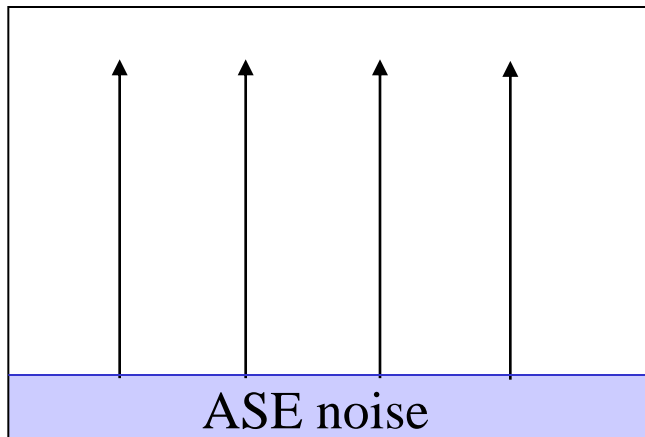


# Presentation Outline

- Introduction to **Parametric Gain (PG)** effects in fibers
- The potential impact of PG in fiber systems
- Drawbacks of the Gaussian approximation
- The **Karhunen-Loève series expansion (KLSE) method**:
  - Brief explanation of the technique and its features
  - Examples: a single span link and a long-haul link
  - System impact of parametric gain
- A possible application (semi-analytical techniques)
- Conclusions

# Parametric Gain (PG)

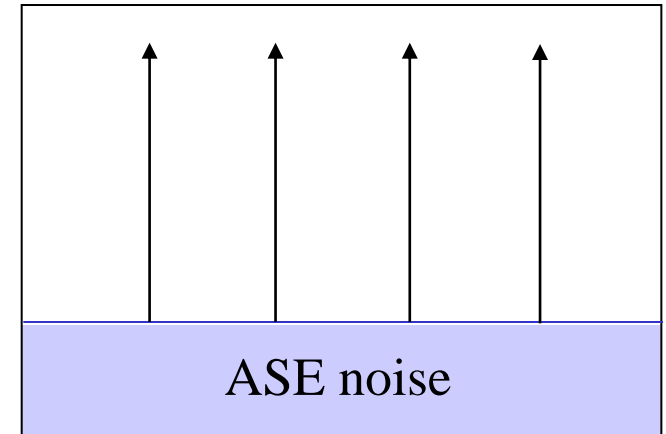
Optical signal at fiber input



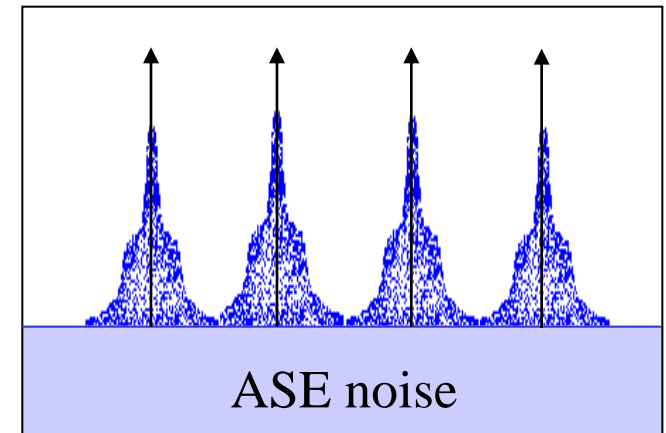
*without PG*



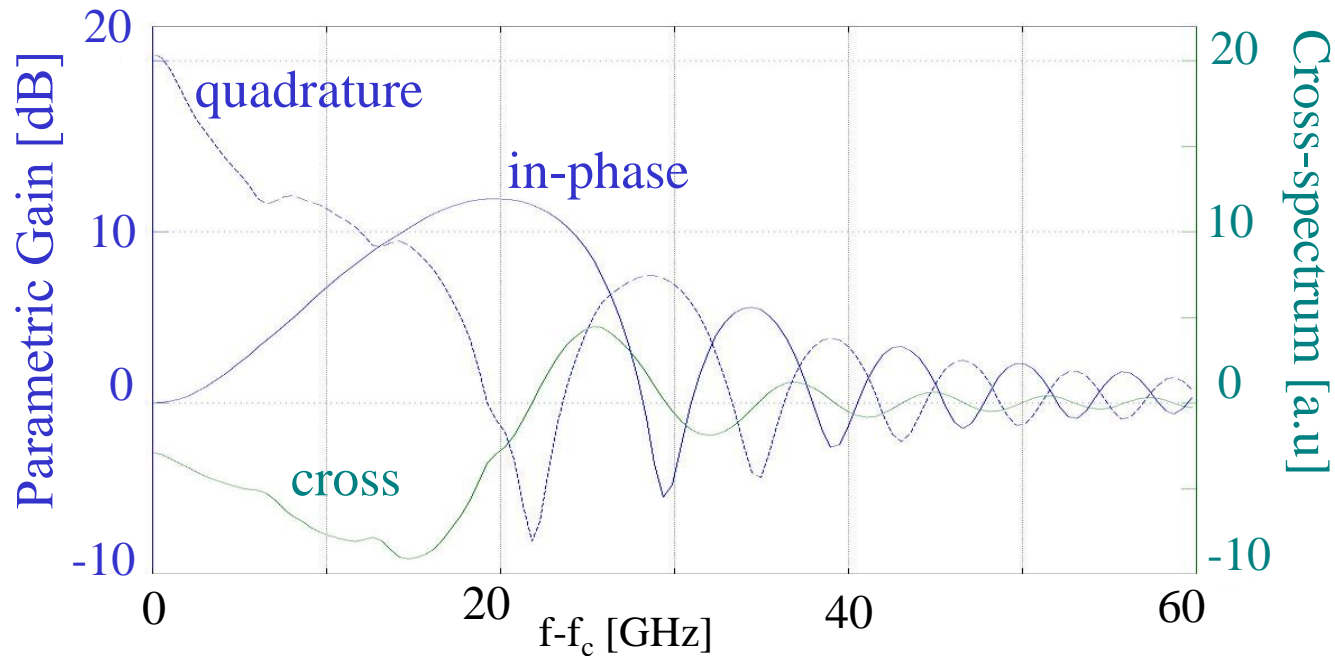
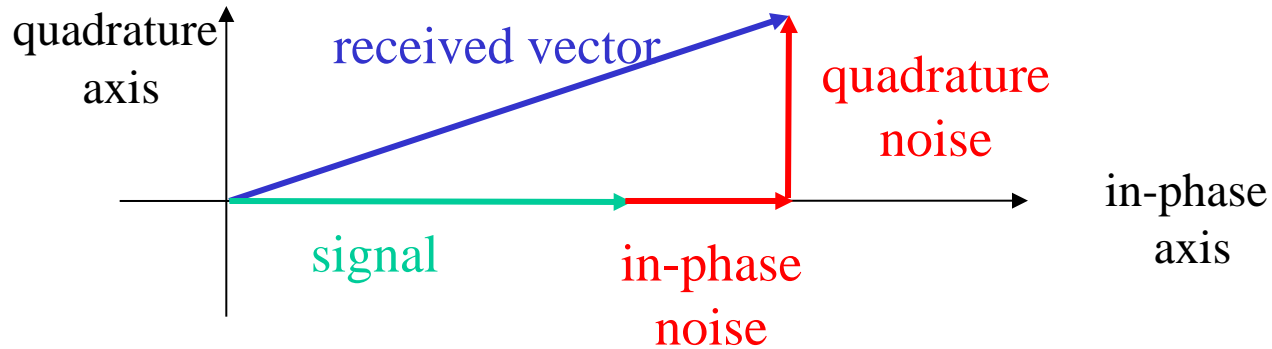
Optical signal at fiber output



*with PG*



# In-phase and quadrature noise components



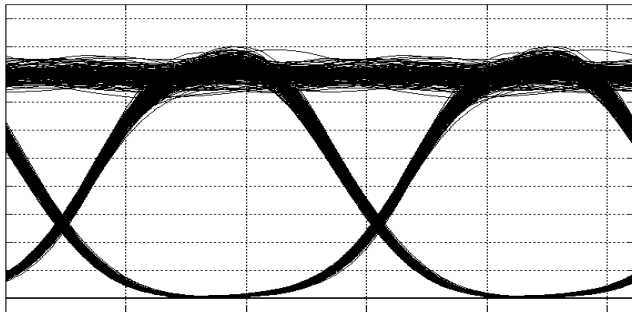
# Potential impact of PG in fibers (\*)

- ASE noise enhancement around the optical carrier, which cannot be filtered out at the receiver
- Pump depletion due to the power transfer to ASE noise

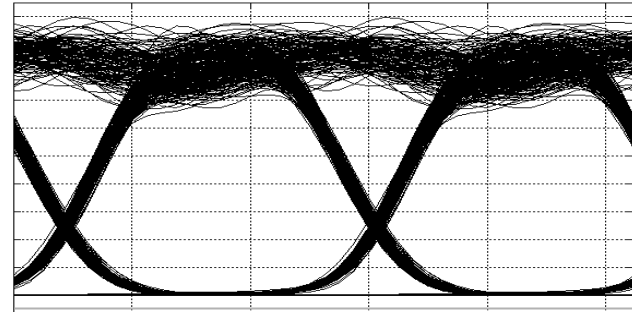


## SYSTEM PERFORMANCE DEGRADATION

without PG



with PG



(\*) R.Hui et al., “Nonlinear amplification of noise in fibers with dispersion and its impact in optically amplified systems”, IEEE Photonics Technology Letters, Vol. 9 no.3 , Mar. 1997 , pp. 392 -394

# Drawbacks of the Gaussian Approximation

- Evaluation of mean and standard deviation of the decision variable for a transmitted logical “1” ( $\mu_1, \sigma_1$ ) and for a transmitted logical “0” ( $\mu_0, \sigma_0$ ).

- Evaluation of the **Q-parameter**:

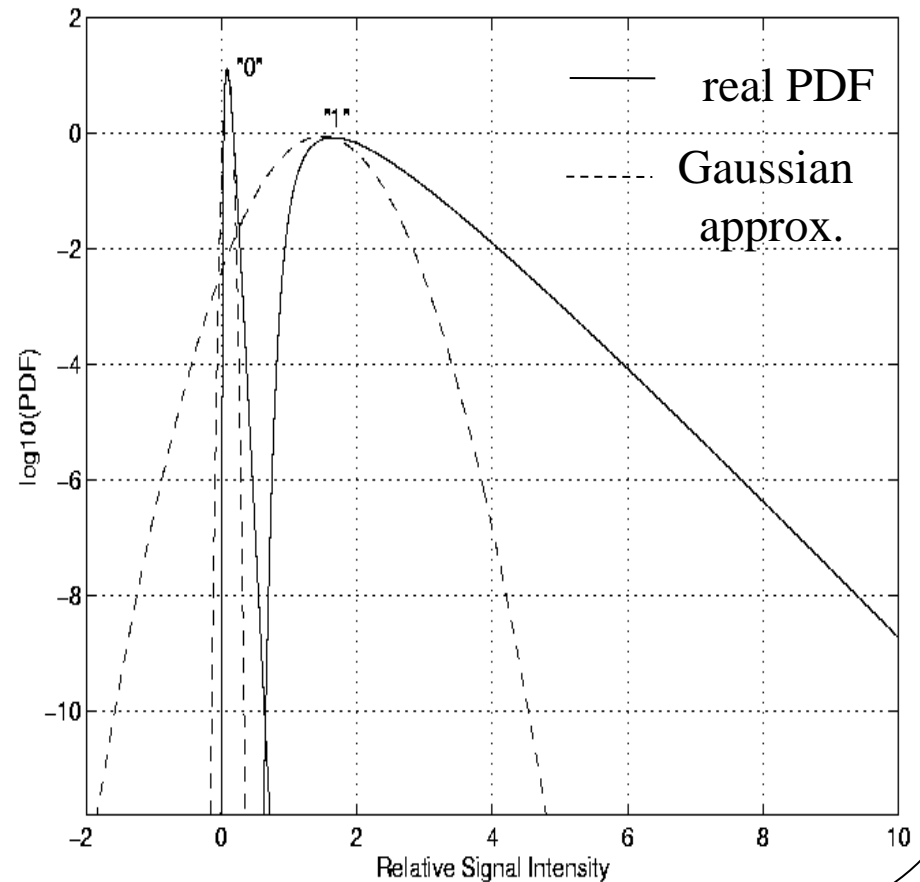
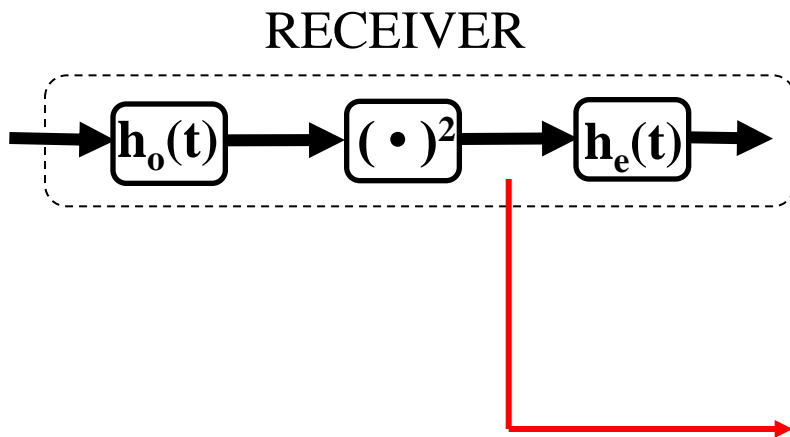
$$Q = \frac{\mu_1 - \mu_0}{\sigma_1 - \sigma_0}$$

- Estimation of the Bit Error Rate (BER):

$$P(e) = \frac{1}{2} \operatorname{erfc} \left( \frac{Q}{\sqrt{2}} \right)$$

# Gaussian approximation accuracy limitations

In a IMDD system the probability density function of the decision variable is strongly non-Gaussian:



# A new technique for BER evaluation based on Karhunen-Loève Series Expansion (\*) (KLSE technique)

- Arbitrary optical and electrical filters
- Exact shape of the noise components power spectra taken into account (evaluated considering PG noise enhancement)
- Correlation between the in-phase and quadrature noise components taken into account

(\*) M.Kac, A.Siegert, “On the theory of noise in radio receivers with square law detectors”, Journal of Applied Physics, vol.18, Apr.1947, pp. 383-397



# Some simplifying hypothesis

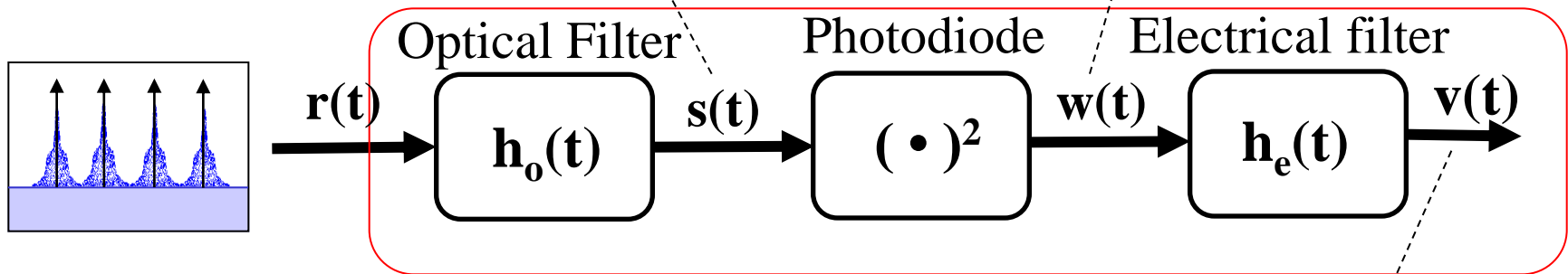
- Absence of intersymbol interference (ISI) at the receiver
- Absence of any transient effect, so that the the complex envelope of the demodulated electric signal, in the absence of noise, is constant in time
- Receiver electrical noise is neglected

# Brief explanation of the KLSE technique

$$s(t) = A + m_P(t) + j m_Q(t)$$

$$w(t) = [A + m_P(t)]^2 + m_Q^2(t)$$

## RECEIVER



$$v(t) = \int_{-\infty}^{+\infty} h_e(\theta) [A + m_P(t - \theta)]^2 d\theta + \int_{-\infty}^{+\infty} h_e(\theta) m_Q^2(t - \theta) d\theta$$

# KLSE fundamental

Noise and signal decomposition on a proper set of orthonormal functions:

$$m_P(t) = \sum_{i=1}^{+\infty} u_i f_i(t), \quad m_Q(t) = \sum_{i=1}^{+\infty} z_i g_i(t), \quad A = \sum_{i=1}^{+\infty} \alpha_i f_i(\theta)$$

Integral equations:

$$\begin{cases} \int_{-\infty}^{+\infty} h_e(\tau) \rho_P(t - \tau) f_i(\tau) d\tau = \lambda_i f_i(t) \\ \int_{-\infty}^{+\infty} h_e(\tau) \rho_Q(t - \tau) g_i(\tau) d\tau = \sigma_i g_i(t) \end{cases}$$

Fundamental property:  $E\{u_i u_j\} = \lambda_i \delta_{ij}, \quad E\{z_i z_j\} = \sigma_i \delta_{ij}$

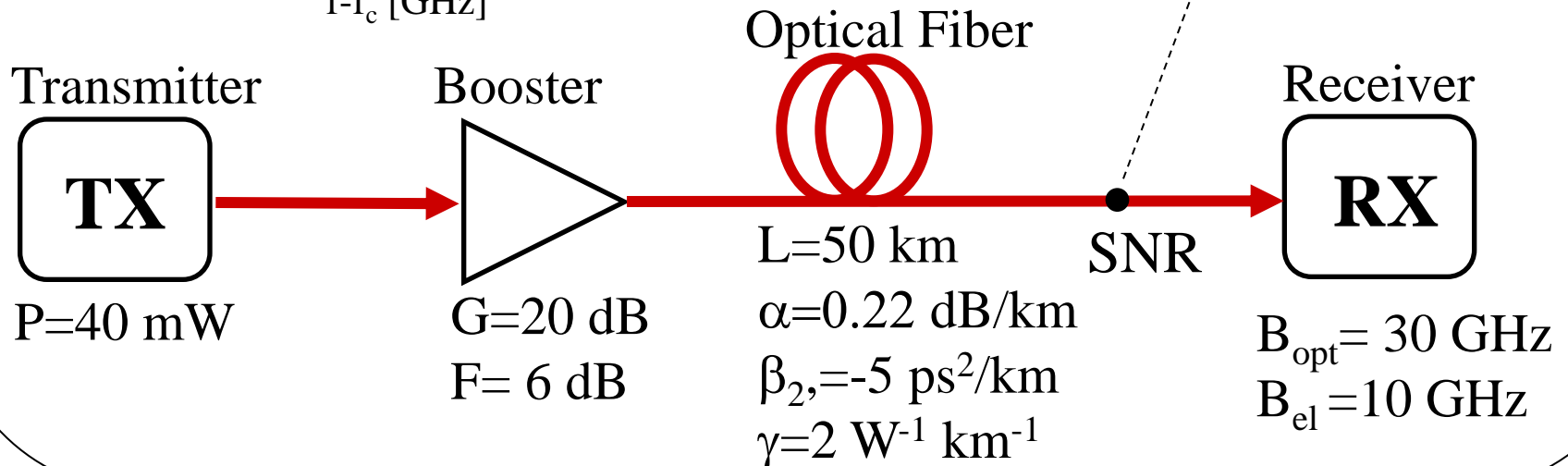
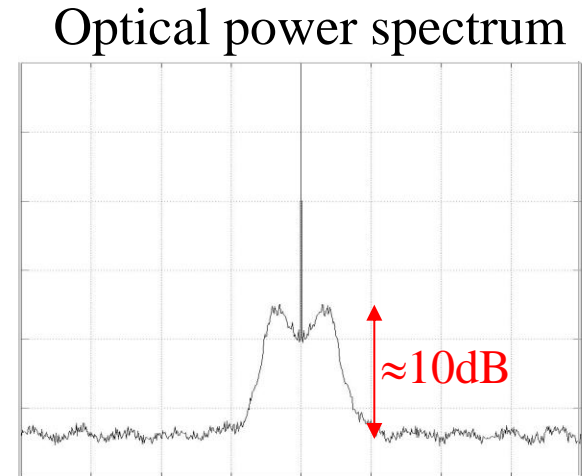
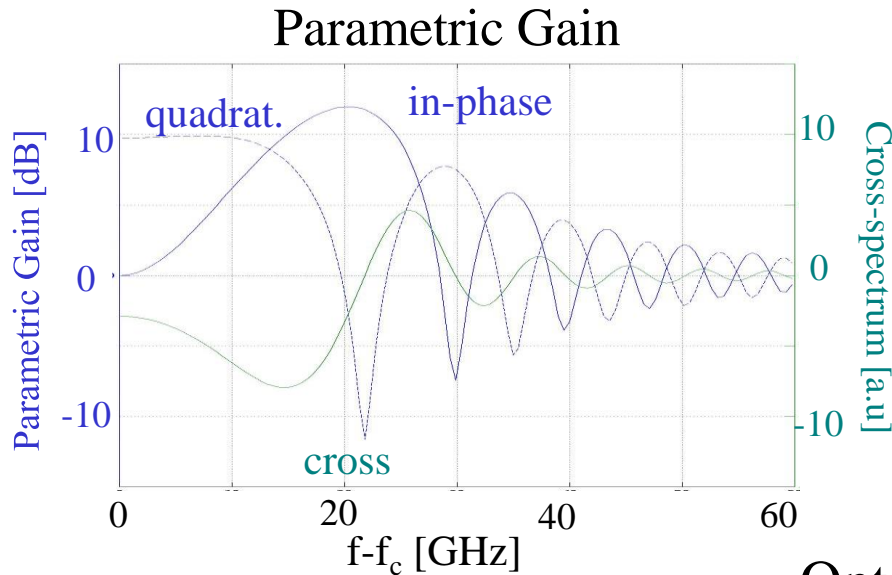
It can be shown that the decision variable can be written as a quadratic form:

$$v = \underline{x}^T \cdot \underline{x}$$

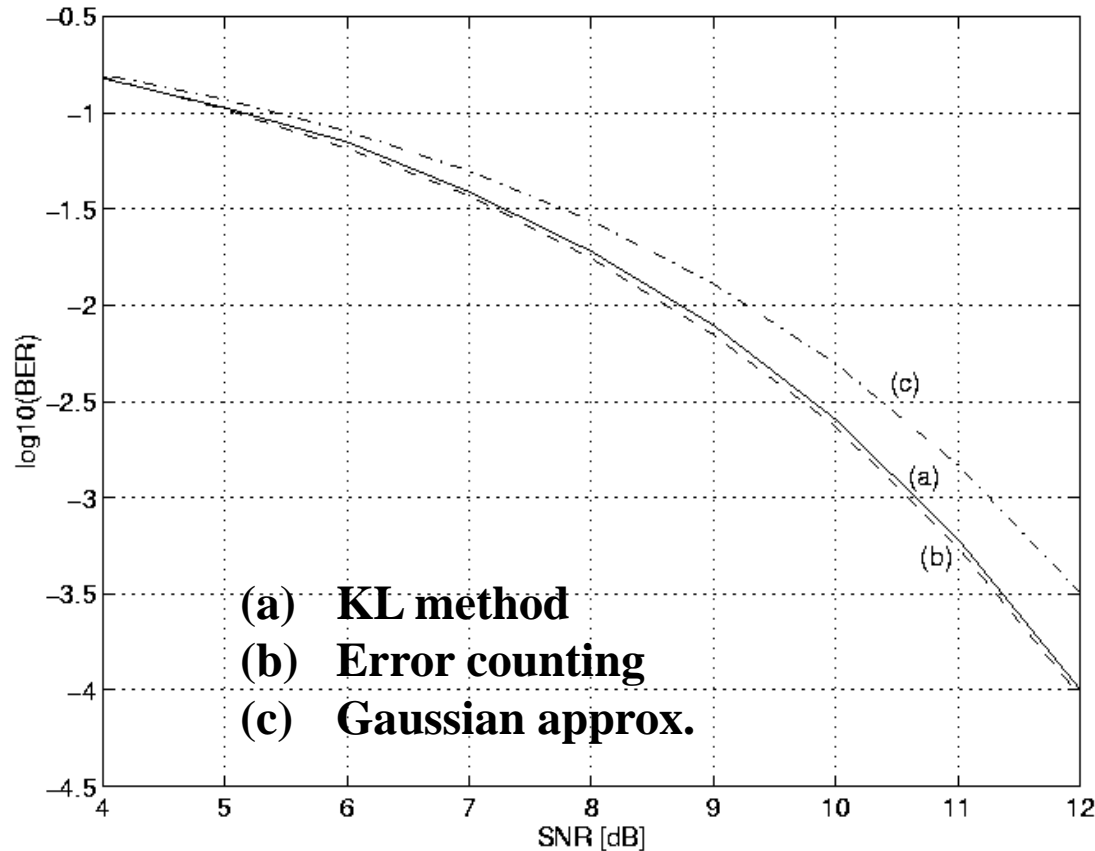
where:  $\underline{x} = [u_1 \dots u_M \ z_1 \dots z_M]^T$        $E\{\underline{x}\} = [\alpha_1 \dots \alpha_M \ 0 \dots 0]^T$

$$E\{\underline{x}^T \cdot \underline{x}\} = \begin{bmatrix} \lambda_1 & 0 & \dots & 0 & E[u_1 z_1^*] & E[u_2 z_1^*] & \dots & E[u_M z_1^*] \\ 0 & \lambda_2 & \dots & 0 & E[u_1 z_2^*] & E[u_2 z_2^*] & \dots & E[u_M z_2^*] \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & \vdots & \vdots & \lambda_M & E[u_1 z_M^*] & E[u_2 z_M^*] & \dots & E[u_M z_M^*] \\ E[u_1^* z_1] & E[u_2^* z_1] & \dots & E[u_M^* z_1] & \sigma_1 & 0 & \dots & 0 \\ E[u_1^* z_2] & E[u_2^* z_2] & \dots & E[u_M^* z_2] & 0 & \sigma_2 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots \\ E[u_1^* z_M] & E[u_2^* z_M] & \dots & E[u_M^* z_M] & 0 & \vdots & \vdots & \sigma_M \end{bmatrix}$$

# Validation of KLSE method by error counting



# Comparison between KL method and Gaussian approximation



- Curves (a) and (b) differ for less than 0.05 dB
- The distance between curve (b) and curve (c) is more than 0.5 dB

# A long-haul compensated link

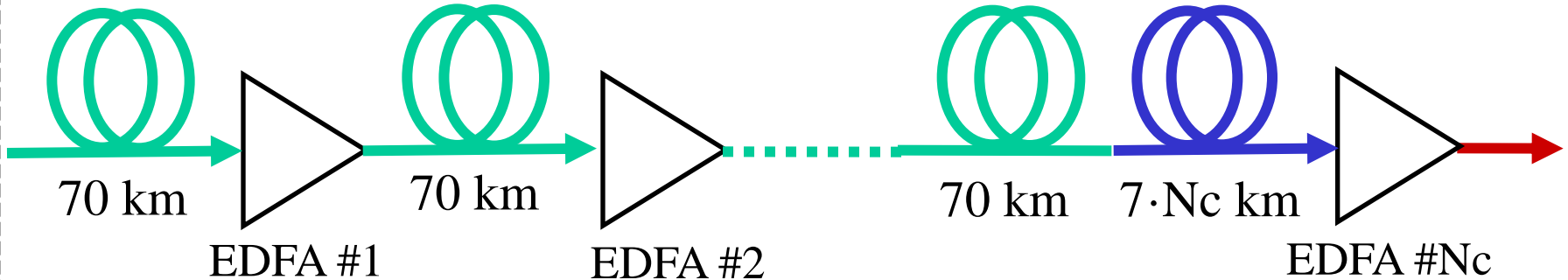
Transmitter



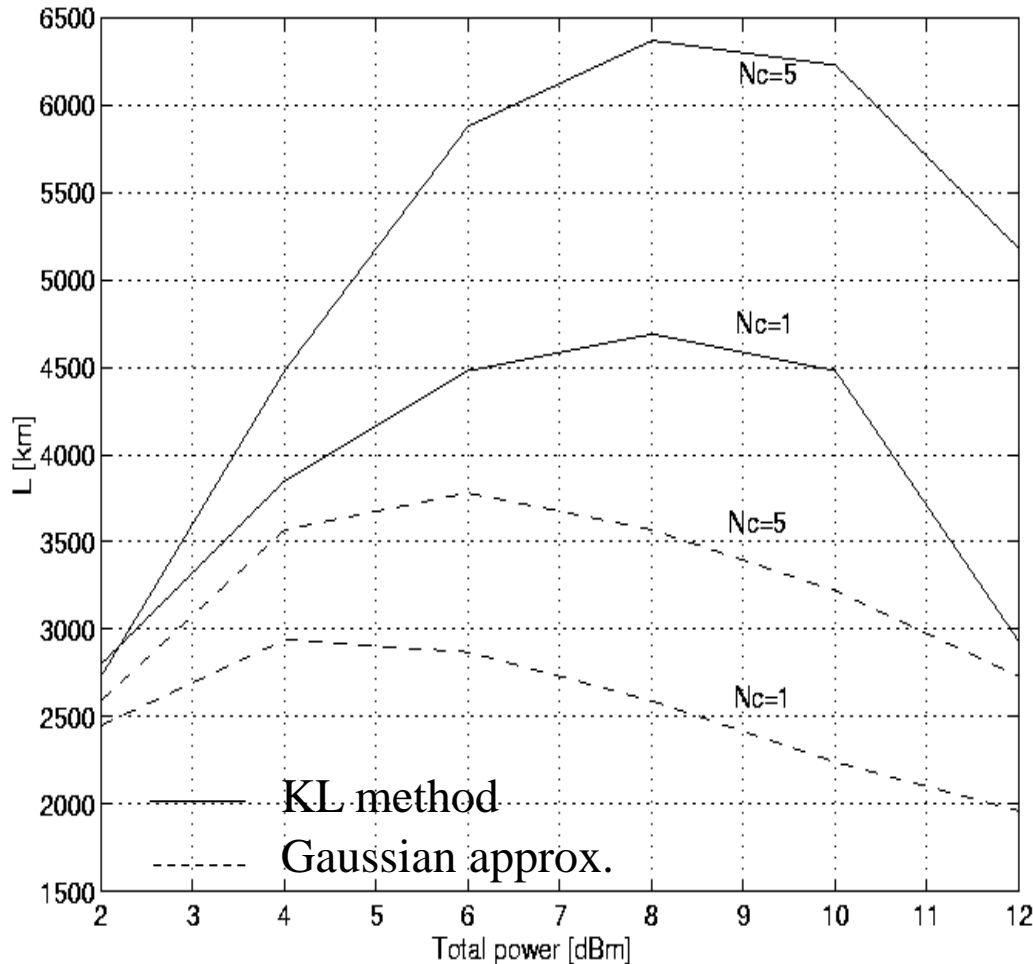
Receiver

DSF:  $\beta_2 = +1 \text{ ps}^2/\text{km}$

DSF:  $\beta_2 = -10 \text{ ps}^2/\text{km}$



# System impact of Parametric Gain



- The maximum reachable distance in presence of Parametric Gain is 6,500 km (11,500 km in linearity)
- Gaussian approximation is too pessimistic whenever PG becomes relevant
- Gaussian approximation also fails in optimizing the transmitted power



# Possible application of KLSE technique

Rapid evolution of optical systems



Need of software simulation tools for performance evaluation

Intrinsic difficulties:

- very low BER → error counting is impractical
- complexity of simulations → few hundreds of bit takes very long time

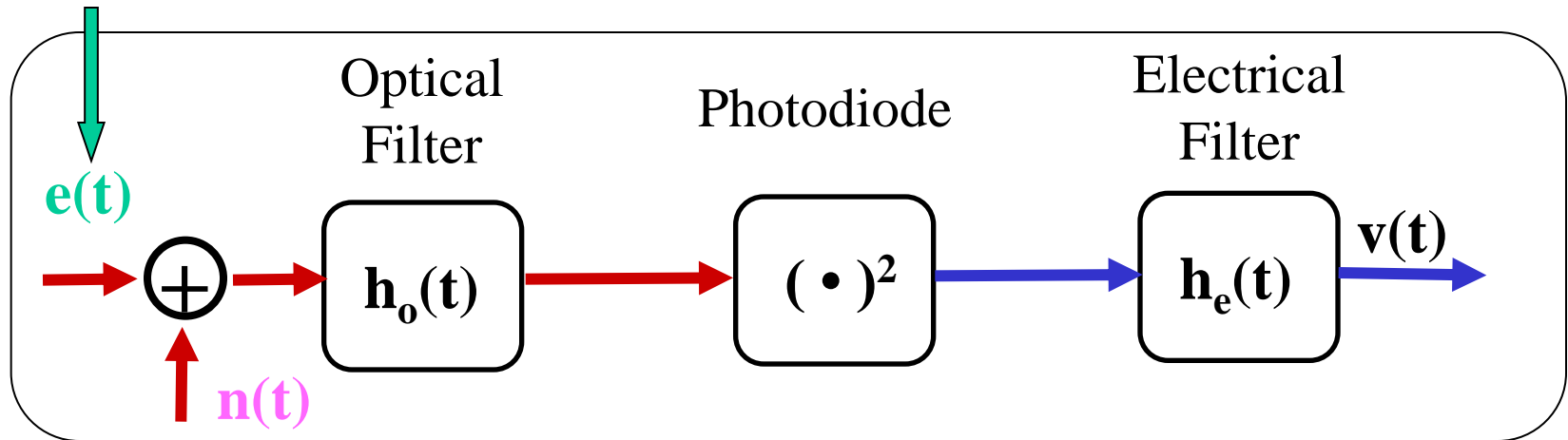


Need for semi-analytical techniques (\*)

(\*) G. Bosco and R. Gaudino, “Towards new semi-analytical techniques for BER estimation in optical system simulation”, NFOEC (National Fiber Optic Engineers Conference), 27-31 August 2000, Denver, Colorado.

# Description of the semi-analytical technique

SIMULATION



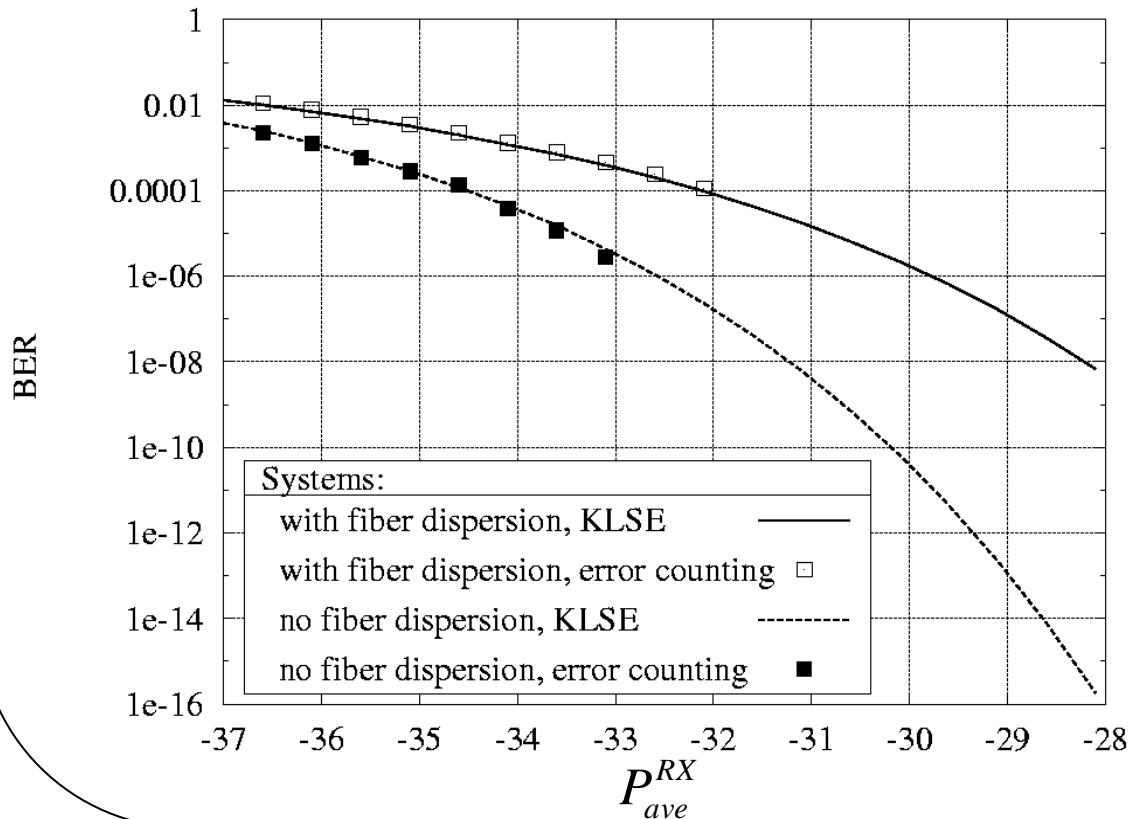
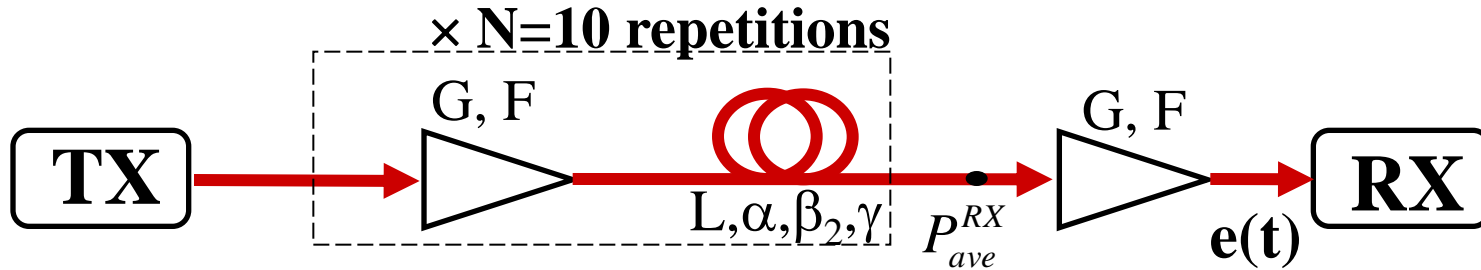
AWGN (analytically evaluated)

It can be shown that  $v(t)$  can still be written as a quadratic form:

$$v(t) = \underline{x}(t)^T \cdot \underline{x}(t)$$

but now the parameters (e.g.  $\lambda_i$ ,  $\sigma_i$ ) depend on time.

# Validation of the method



## Fiber Parameters:

$L=50$  Km

$\alpha = 0.2$  dB/km

$\beta_2 = -2$  ps<sup>2</sup>/km

## Amp. parameters:

$G=10$  dB

$F= 4.5$  dB

# Conclusions

- **Parametric Gain (PG)** is a nonlinear phenomenon which enhances ASE noise in fiber propagation and can be very detrimental in amplified optical systems  
The widely used Gaussian approximation fails in estimating system performance whenever PG becomes relevant
- The **KLSE technique** is a powerful instrument to analyze the impact of PG on optical systems, since it takes into account the exact shape of the noise components power spectra
- Moreover, it can be easily extended to obtain a valid semi-analytical method to evaluate bit-error rate in optical system simulations