A novel analysis of the impact of Parametric Gain on WDM systems

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Outline

- Introduction.
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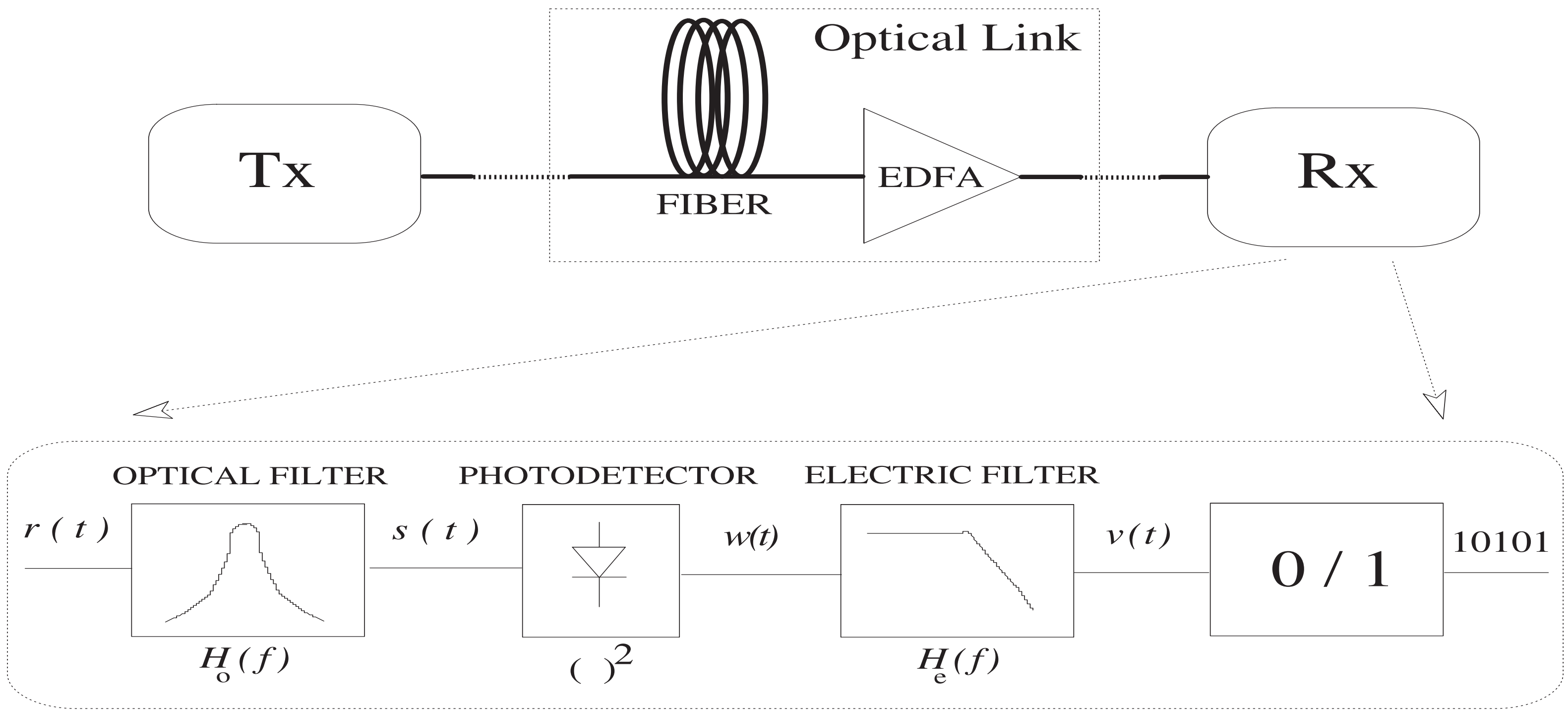
Final purpose

Exact BER evaluation in a standard optical direct-detection receiver where ASE noise is the prelevant noise source.



Why Karhunen-Loève?

- Presence of a quadratic element (the photodetector) $\Rightarrow$ highly non-Gaussian noise at the output of the receiver (*);
- presence of non-linear effects such as PG $\Rightarrow$ non-white received ASE noise, with correlated real and imaginary components (**).

(*) this problem has been solved in the literature under the assumption that received ASE is a white Gaussian noise, using a theory based on the Karhunen-Loève (KL) expansion;

(**) our technique allows to evaluate the BER in presence of an arbitrary spectrally shaped received noise.

Karhunen-Loève expansion (1)

Let $z(t)$ denote a possibly nonstationary random process with zero mean and autocorrelation function $\rho(t,\tau)$. $z(t)$ can be written as:

$$z(t) = \sum_n z_n f_n(t)$$

where:

$$\int_{-\infty}^{+\infty} h_e(t) f_i(t) f_j^*(t)\,\mathrm{d}t = \delta_{ij}, \quad z_n = \int_{-\infty}^{+\infty} h_e(t) z(t) f_n^*(t)\,\mathrm{d}t$$

Karhunen-Loève expansion (2)

It can be shown that, if the set of orthonormal functions $\{f_n(t)\}$ are the eigenfunction of the integral equation:

$$\int_{-\infty}^{+\infty} h_e(\tau)\rho(t)f_n(\tau)\,\mathrm{d}\tau = \lambda_n f_n(t)$$

then the coefficients $\{z_n\}$ are mutually incorrelated Gaussian random variables with variance λ_n.

Some simplifying hypothesys

1) An arbitrary ASE noise spectral density at the receiver, which will be evaluated taking into account the PG noise enhancement;

2) absence of intersymbol interference (ISI) at the receiver;

3) absence of any transient effect, so that the demodulated electrical signal, in the absence of noise, is constant in time (in the complex envelope representation);

4) receiver electrical noises, both thermal or shot, are not taken into account.

The received optical signal

The complex envelope representation of the received optical signal after the optical filter is:

$$s(t) = A + m_P(t) + \jmath\, m_Q(t)$$

where:

- $m_P(t)$ is the in-phase noise component with autocorrelation function $\rho_P(\tau)$;
- $m_Q(t)$ is the quadrature noise component with autocorrelation function $\rho_Q(\tau)$;
- the mutual correlation beetween $m_P(t)$ and $m_{PQ}(t)$ is $\rho_{PQ}(\tau)$.

Karhunen-Loève expansion

After the electrical filter:

$$v(t) = \int_{-\infty}^{+\infty} h_e(\theta)[A + m_P(t-\theta)]^2 \, \mathrm{d}\theta \; + \int_{-\infty}^{+\infty} h_e(\theta) m_Q^2(t-\theta) \, \mathrm{d}\theta$$

We expand $m_P(t)$ and $m_Q(t)$ in the following Karhunen-Loève series:

$$\begin{aligned} m_P(t) &= \sum_{i=1}^{+\infty} u_i f_i(t) \\ m_Q(t) &= \sum_{i=1}^{+\infty} z_i g_i(t) \end{aligned}$$

where:

$$\int_{-\infty}^{+\infty} h_e(\tau)\rho_P(t-\tau) f_i(\tau) \, \mathrm{d}\tau = \lambda_i f_i(t)$$
$$\int_{-\infty}^{+\infty} h_e(\tau)\rho_Q(t-\tau) g_i(\tau) \, \mathrm{d}\tau = \sigma_i g_i(t)$$

The decision variable (1)

Change of variables:

$$\begin{aligned} m_P(t-\theta) &= \sum_{i=1}^{+\infty} u_i' f_i(\theta) \\ m_Q(t-\theta) &= \sum_{i=1}^{+\infty} z_i' g_i(\theta) \end{aligned}$$

where u_i' and z_i' are random variables with the same properties as u_i and v_i, due to the stationarity of the input noise random process.
Signal carrier expansion:

$$A = \sum_{i=1}^{+\infty} \alpha_i f_i(\theta)$$

It can be shown that:

$$v = \sum_{i=1}^{+\infty} (\alpha_i + u_i)^2 + \sum_{i=1}^{+\infty} z_i^2 \quad .$$

The decision variable (2)

The decision variable v can be rewritten as a quadratic form:

$$v = \underline{x}^T \cdot \underline{x} = \sum_{i=1}^{2M} x_i^2$$

where $\underline{x}$ is a vector random variable with:

$$\begin{aligned} E\{\underline{x}\} &= \underline{m} = [\alpha_1 \ldots \alpha_M \;\; 0 \ldots 0]^T \\ E\{\underline{x} \cdot \underline{x}^T\} &= \underline{\underline{R}} \end{aligned}$$

Diagonalization of $\underline{\underline{R}}$:

$$\underline{y} = \underline{\underline{P}}^T \cdot \underline{x} \qquad \text{with} \qquad \underline{\underline{P}}^T \cdot \underline{\underline{R}} \cdot \underline{\underline{P}} = \text{diag}\{\psi_i\}$$

The Moment Generating Function (MGF)

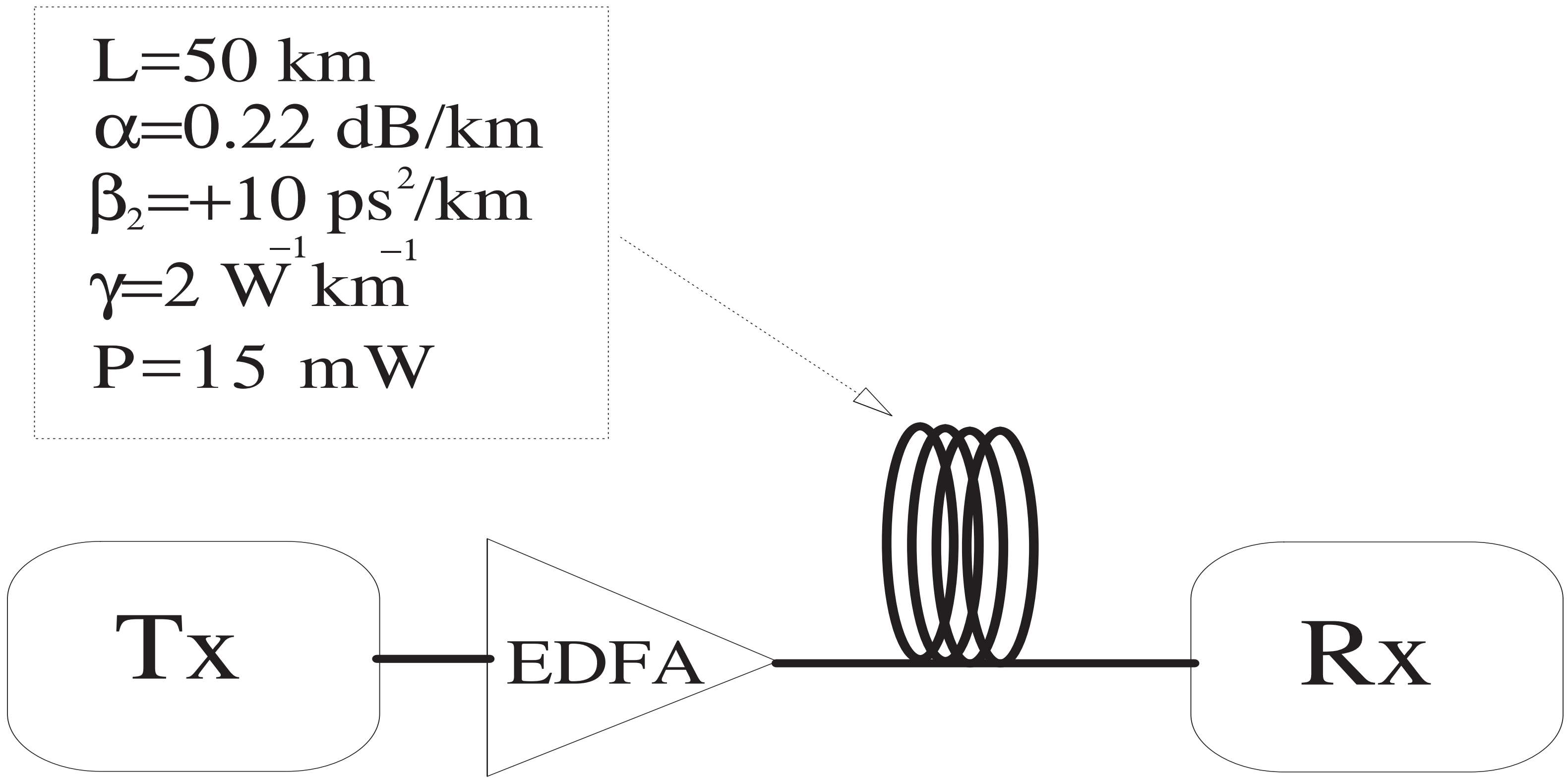
$$v = \underline{y}^T \cdot \underline{y} = \sum_{i=1}^{2M} y_i^2 \quad \text{with} \quad \begin{array}{rcl} E\{\underline{y}\} & = & \underline{\underline{P}}^T \cdot \underline{m} \\ E\{\underline{y} \cdot \underline{y}^T\} & = & \mathrm{diag}\{\psi_i\} \end{array}$$

The MGF Φ_v of the random variable v is:

$$\Phi_v(z) = E\{\mathrm{e}^{-zv}\} = \prod_{i=1}^{2M} \frac{\exp\left\{\frac{-\delta_i^2 z}{1+2\psi_i z}\right\}}{\sqrt{(1+2\psi_i z)}}$$

where the δ_i's are the elements of $\underline{\underline{P}}^T \cdot \underline{m}$ and the ψ_i's are the elements of $\underline{\underline{\psi}}$, which are equal to the eigenvalues of $\underline{\underline{R}}$.

A single-span link

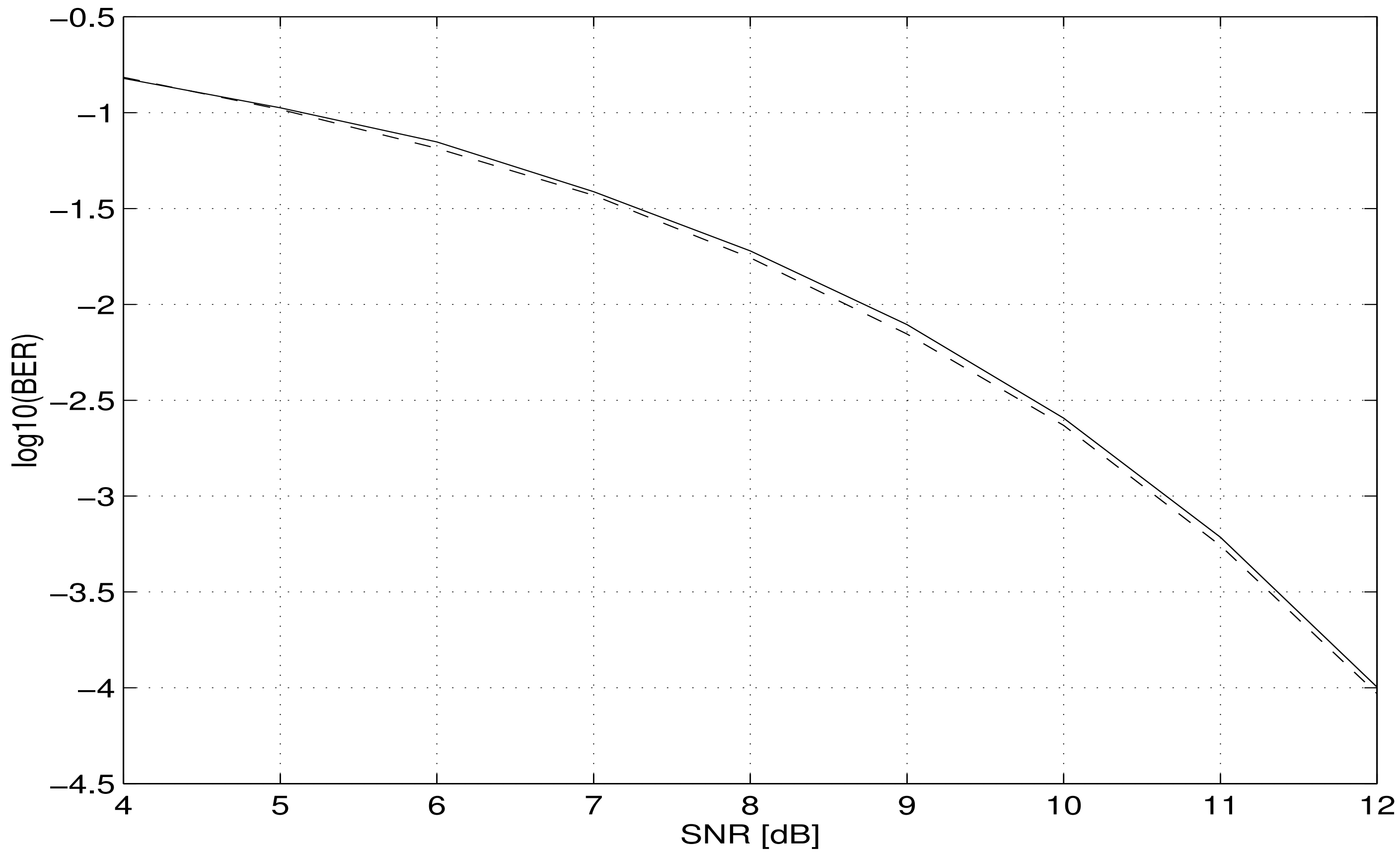


Optical filter bandwidth: 25 Ghz

Electrical filter bandwidth: 10 Ghz

Validation of results

Comparison between the KL method (solid line) and simulation by error counting (dashed line).



Q-factor method

$$P(e) = \frac{1}{2}\mathrm{erfc}\left(\frac{Q}{\sqrt{2}}\right)$$

where the Q parameter is defined as:

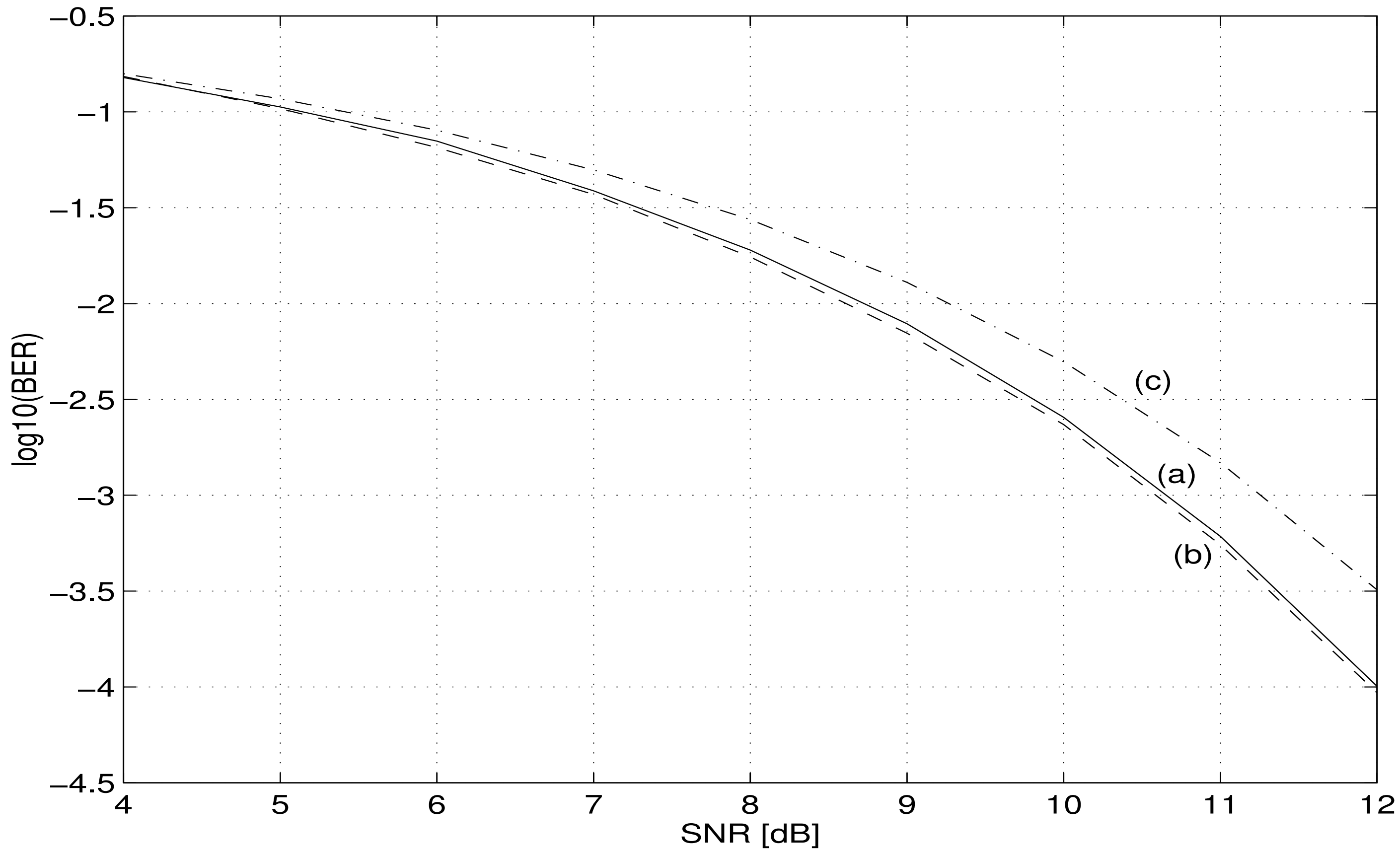
$$Q = \frac{m_1 - m_0}{\sigma_1 + \sigma_0}$$

m_1, σ_1 and m_0, σ_0 are the mean and standard deviation of the decision variable when a logical "1" or "0" are transmitted, respectively.

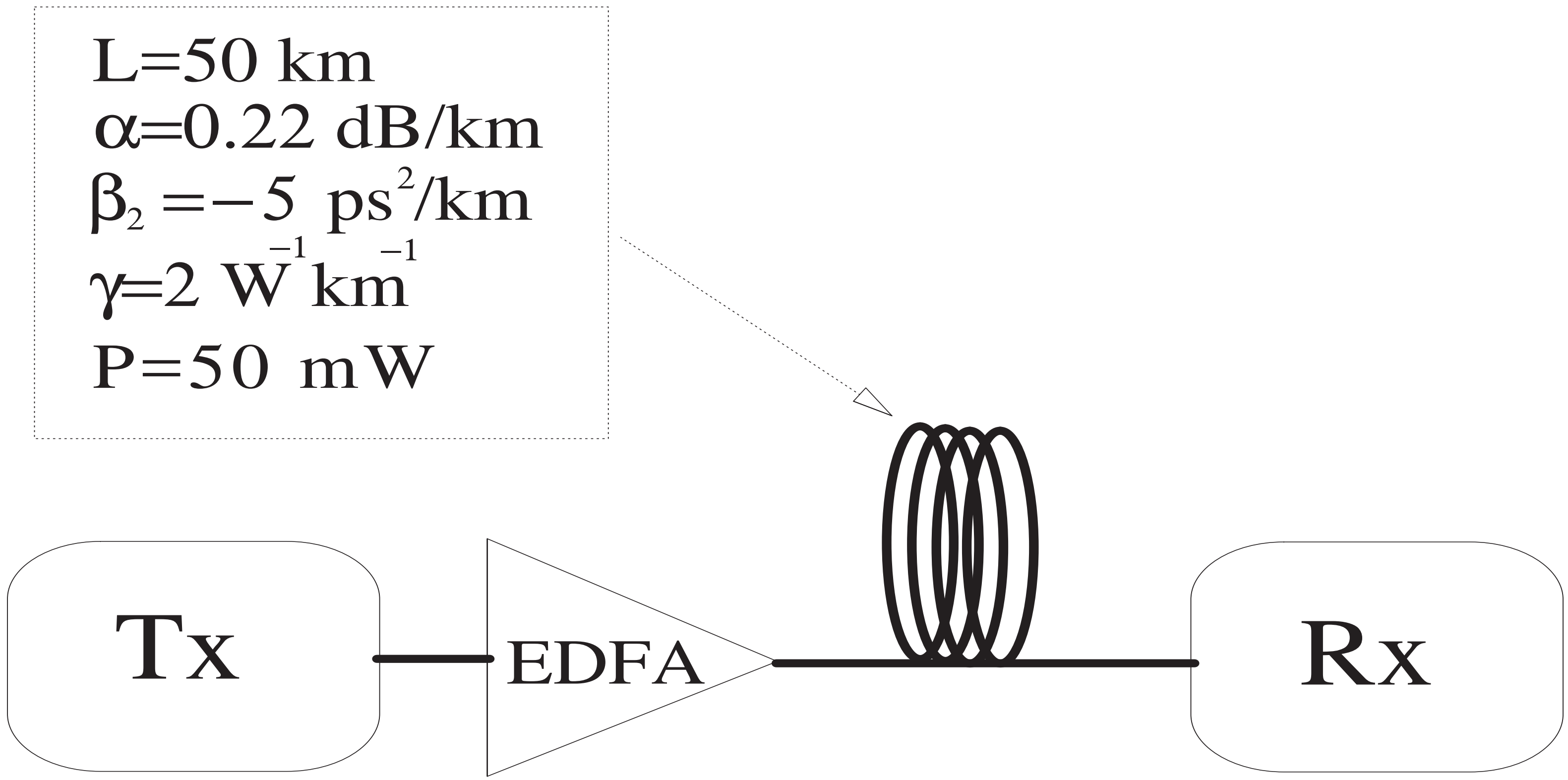
This approximation is extensively used in experiment, theory and simulation of the performance of optical systems.

Comparison

Comparison between the KL method (line (a)), simulation by error counting (line (b)) and Q-factor method (line(c)).



Another single-span link

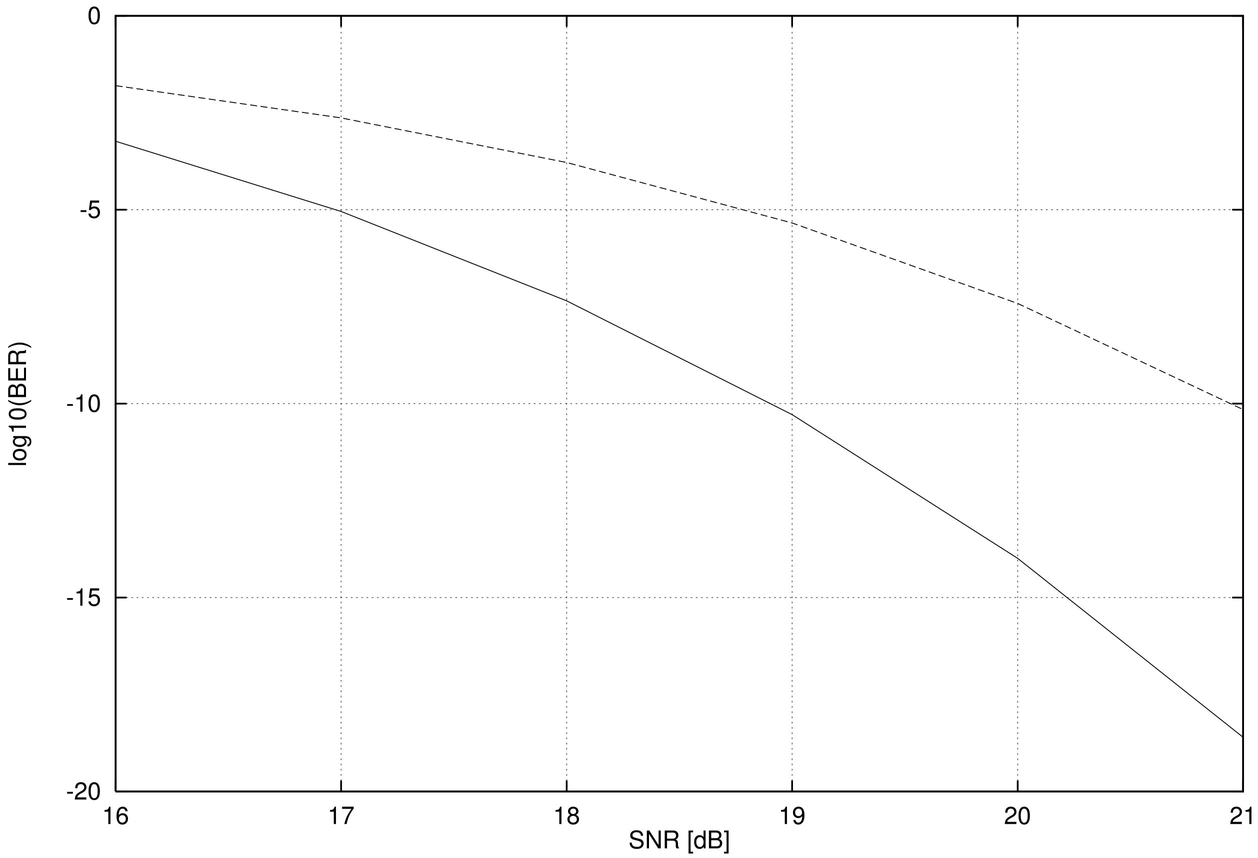


Optical filter bandwidth: 40 Ghz

Electrical filter bandwidth: 10 Ghz

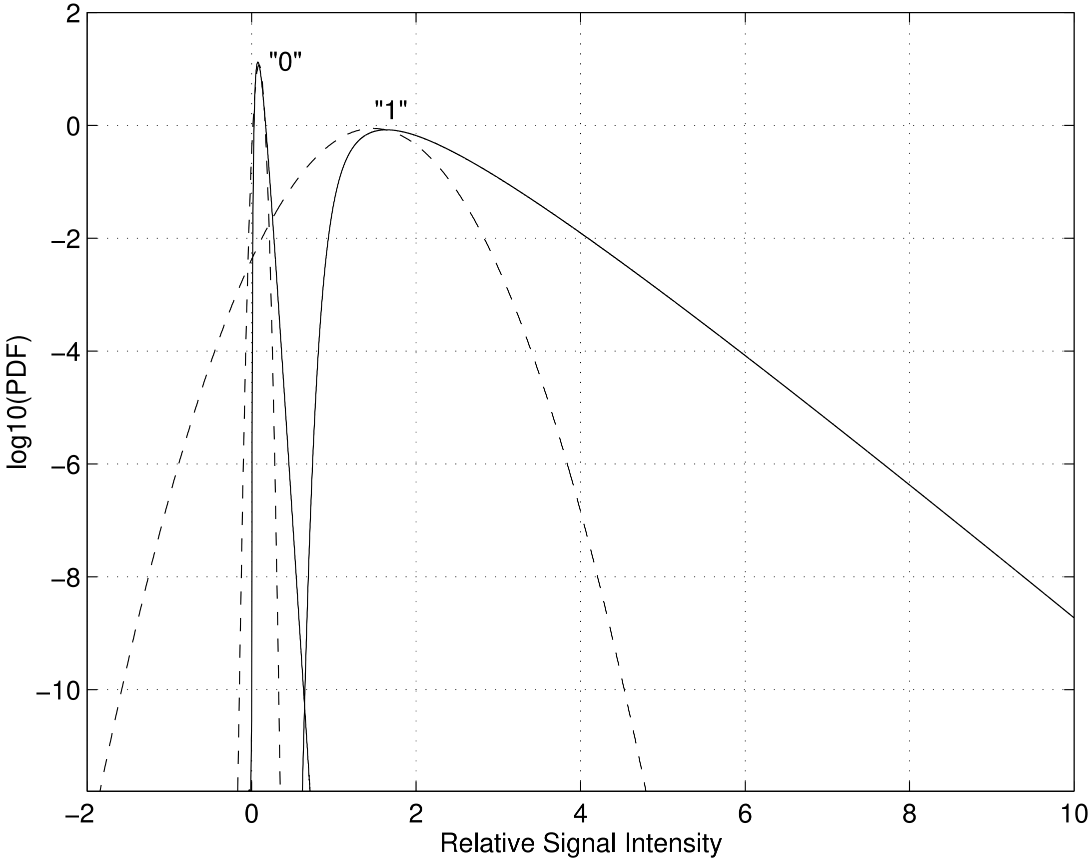
Comparison

Comparison between the KLSE method (solid line) and the Q-factor method (dashed line).



PDF

PDF of the electrical decision signal after photodetection and filtering.



An ideal experiment

Both the in-phase and quadrature noise components are white Gaussian uncorrelated processes:

- In-phase component power spectral density: $N_0/2$;
- quadrature component power spectral density: $G \cdot N_0/2$.

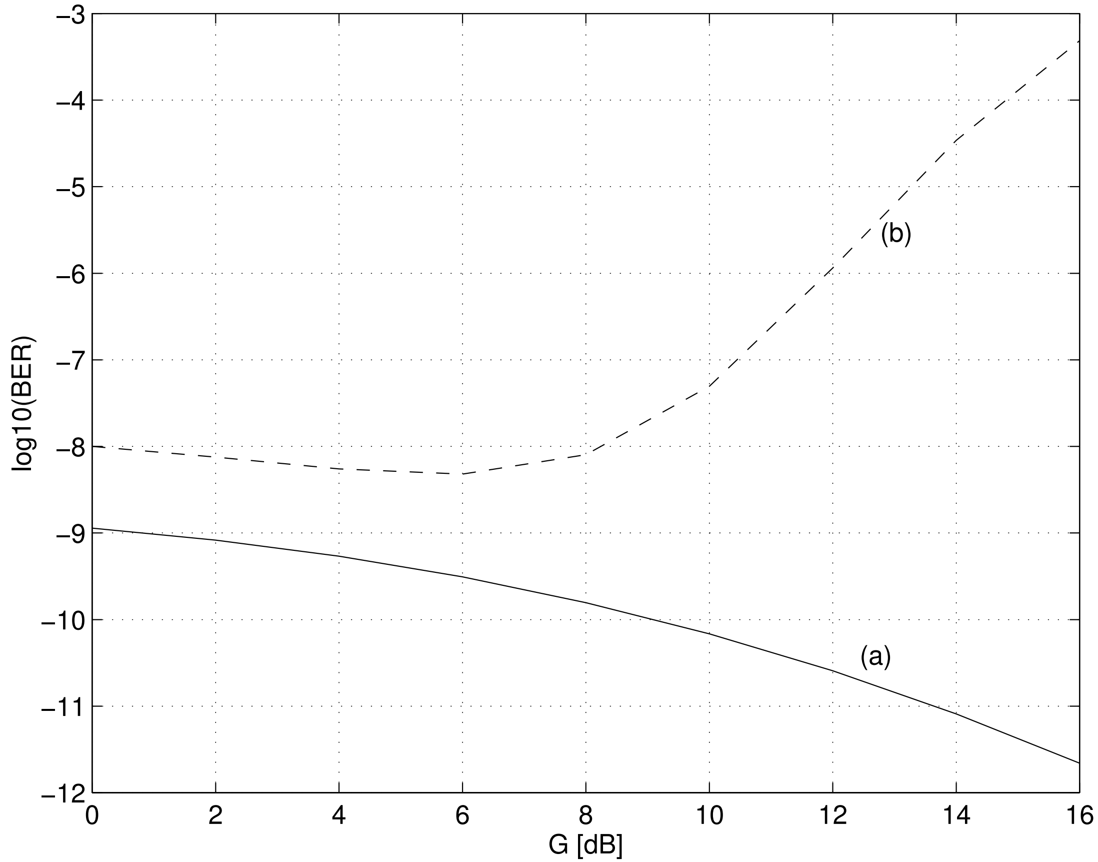
We have considered two situations:

a) pump undepletion;

b) pump depletion (due to the transfer of power from the carrier to noise).

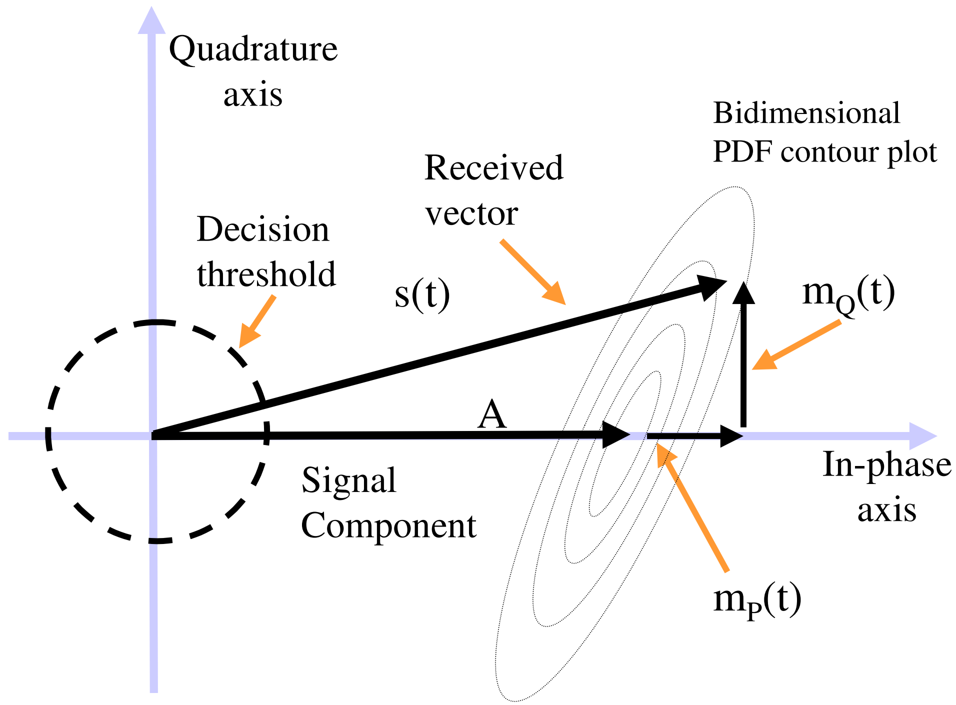
Quadrature noise (pump undepletion)

BER evaluated employing the KL method (solid line) and the Gaussian method (dashed line) as functions of the quadrature noise gain G.



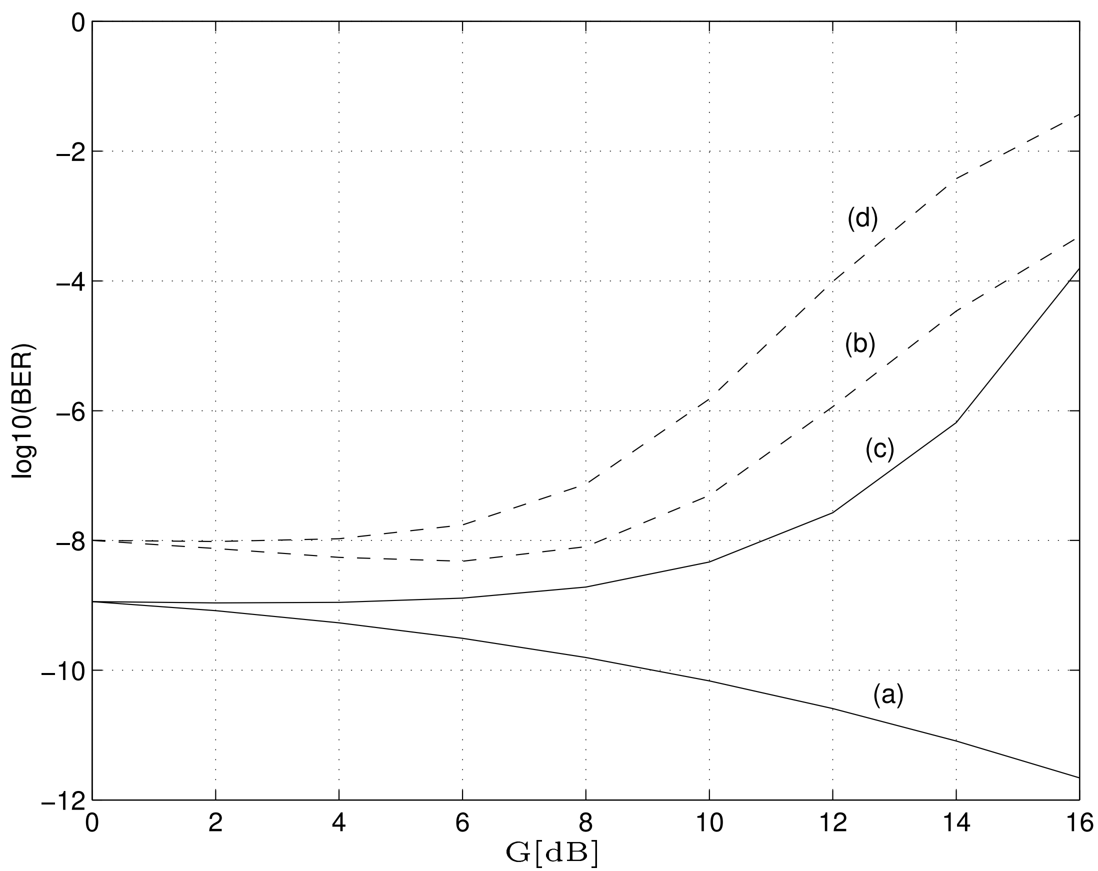
Schematic representation of optical signal

Schematic representation of the received optical signal before photodetection, accounting for signal and noise in the complex plane.

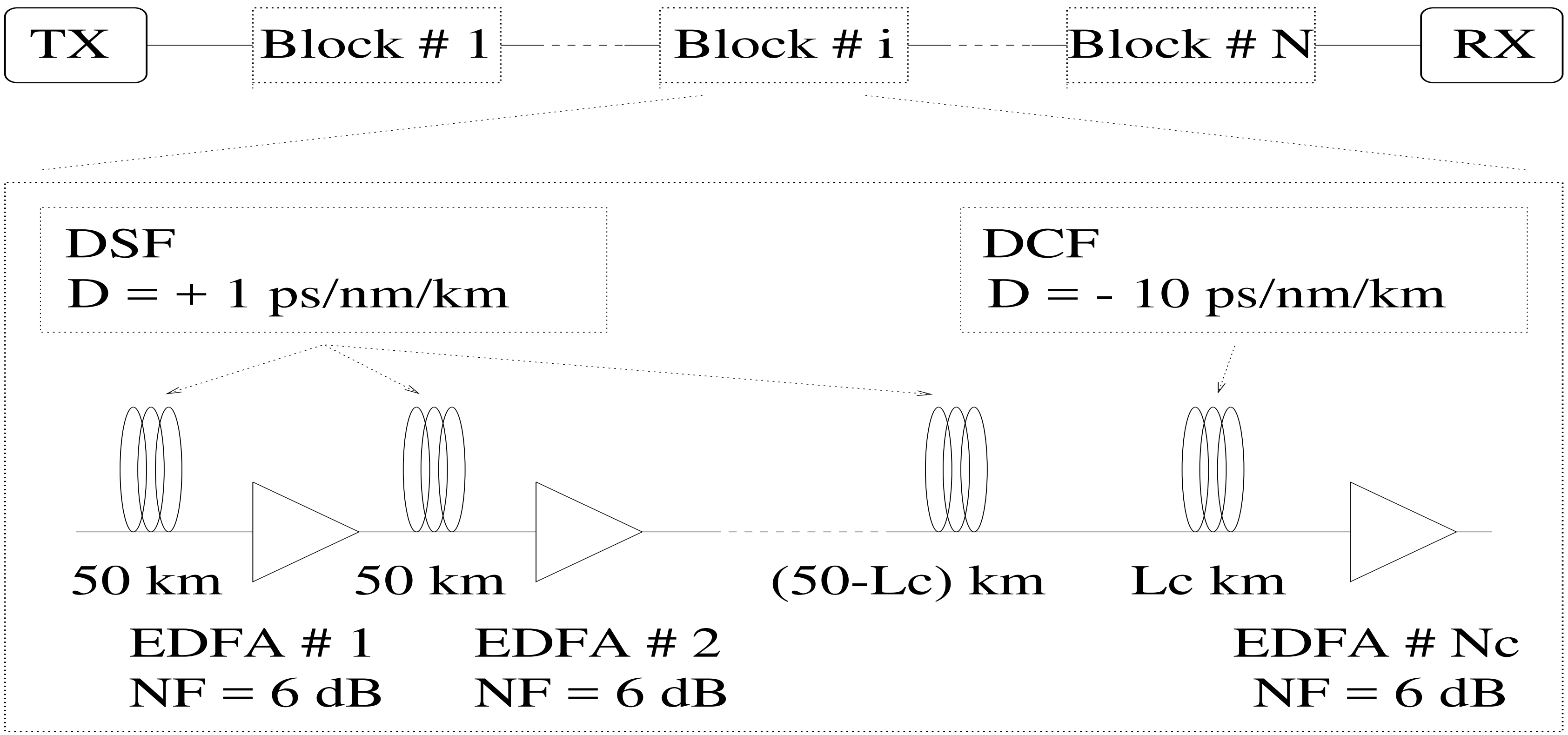


Quadrature noise

BER in the hypothesis of pump undepletion (curves a,b) and in the hypothesis of pump depletion (curves c,d).



A long-haul compensated link



$\alpha = 0.22$ dB/km, $\gamma = 2\ \mathrm{W}^{-1}\mathrm{km}^{-1}$

Optical filter bandwidth: 40 Ghz

Electrical filter bandwidth: 10 Ghz

System impact of PG

Maximum distance giving $P(e) \leq 10^{-12}$ as a function of the total transmitted power, evaluated with KL (solid lines) and Q (dashed lines).