A Novel Update Algorithm in Stokes Space for Adaptive Equalization in Coherent Receivers

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Coherent detection and DSP

High-order modulation

Coherent detection

Digital signal processing

High spectral efficiency transmission over long-haul optical links

Coherent detection and DSP

High-order modulation

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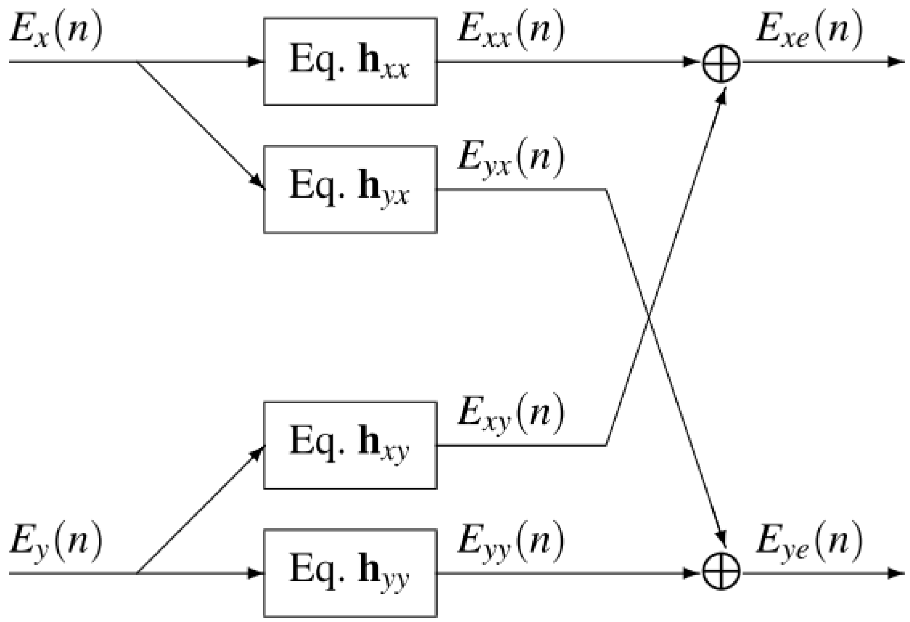
Digital signal processing

High spectral efficiency transmission over long-haul optical links

- Front-end IQ imbalance
- Chromatic dispersion
- PMD
- PDL
- Residual CD
- Filtering effects
- Frequency offset
- Laser phase noise
- Fiber non-linear effects

Adaptive MIMO equalizer

MIMO equalizer



$$E_{xe}(n) = \mathbf{E}_x^T \mathbf{h}_{xx} + \mathbf{E}_y^T \mathbf{h}_{xy}$$

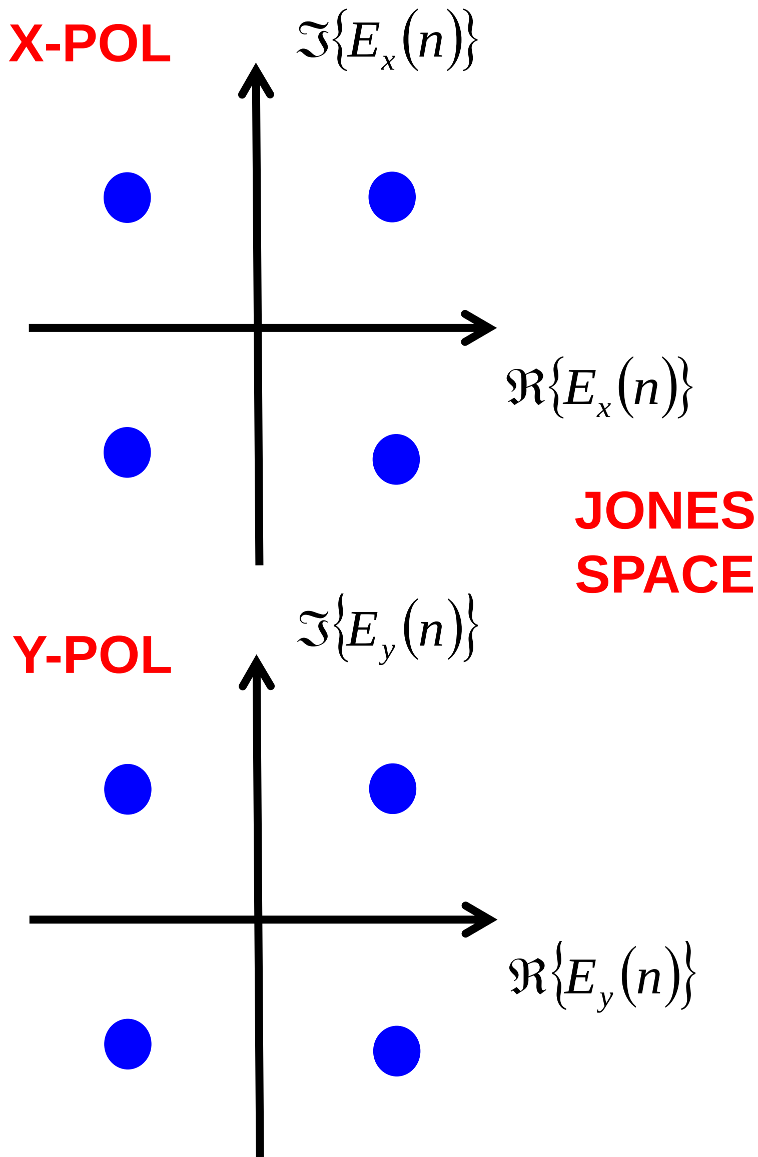
$$E_{ye}(n) = \mathbf{E}_x^T \mathbf{h}_{yx} + \mathbf{E}_y^T \mathbf{h}_{yy}$$

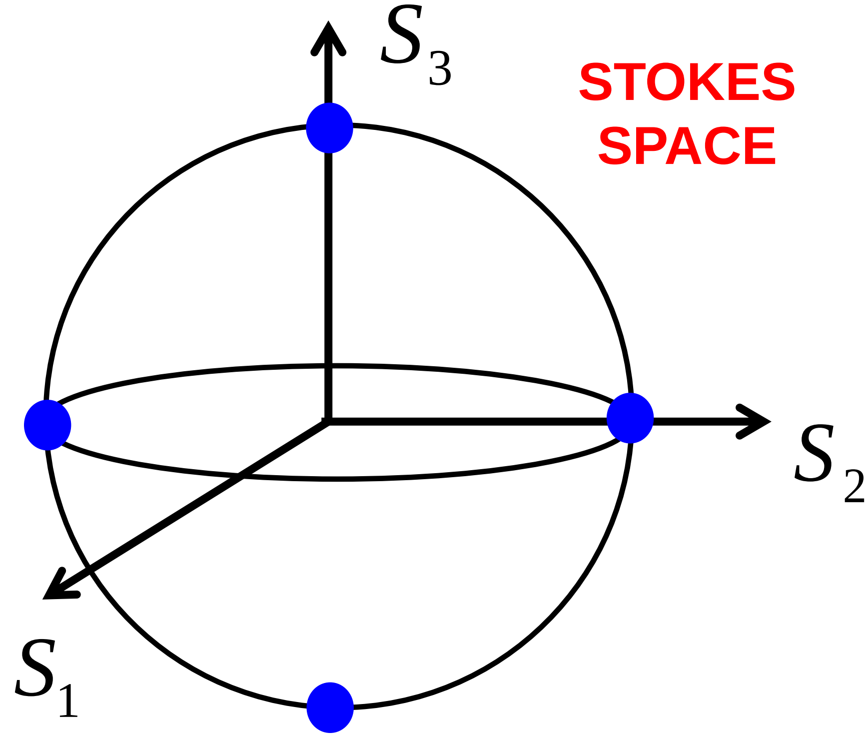
- Standard CMA or LMS algorithms: update of the coefficients based on error signals evaluated on the two-dimensional constellations (separate for the two polarizations)

- **New algorithm: update of the coefficients based on error signal evaluated in the Stokes space**
  - Performance test on a PM-16QAM signal, comparing it to the multi-modulus CMA algorithm

PM-QPSK

$$S_1(n) = |E_x(n)|^2 - |E_y(n)|^2$$
$$S_2(n) = 2\Re\{E_x(n)E_y(n)\}$$
$$S_2(n) = 2\Im\{E_x(n)E_y^*(n)\}$$



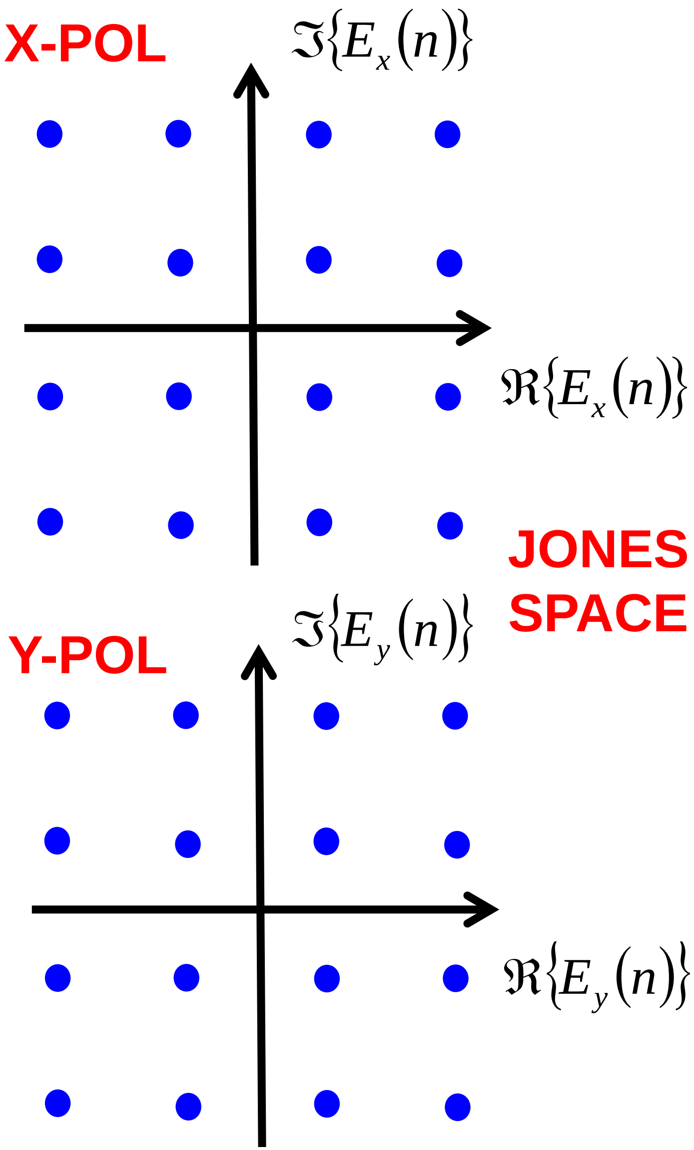


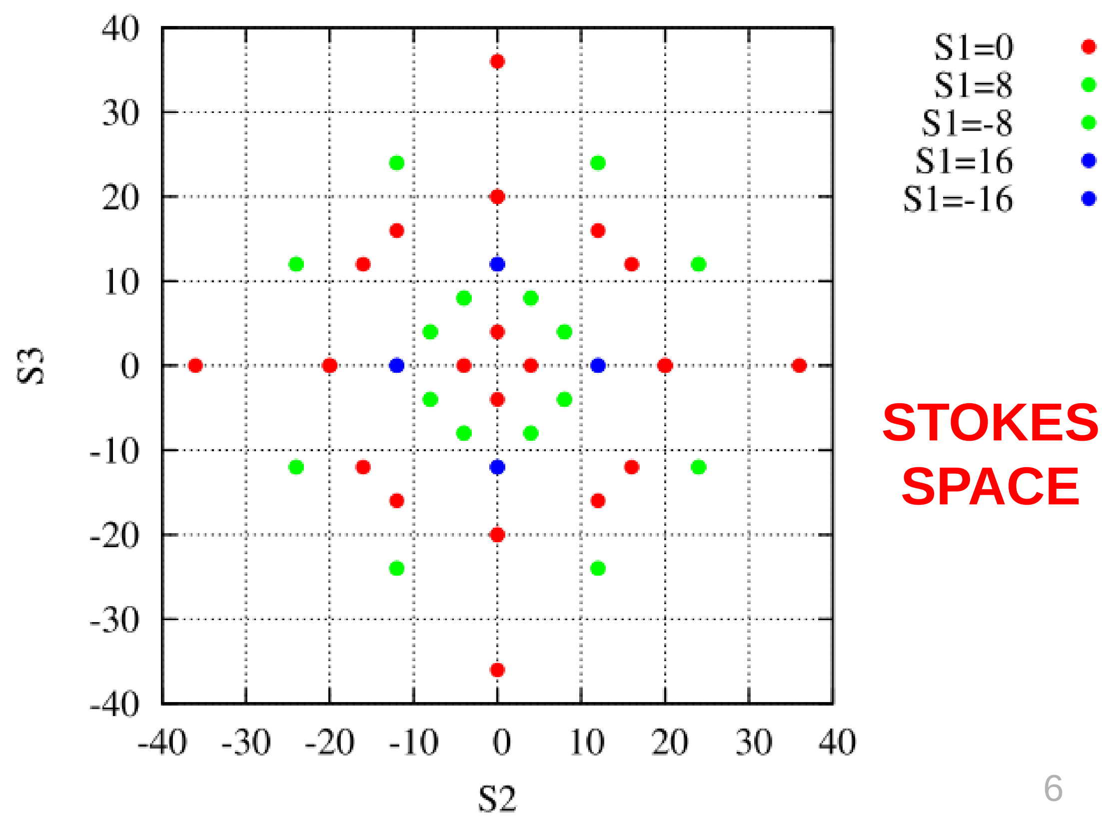
PM-16QAM

$$S_1(n) = |E_x(n)|^2 - |E_y(n)|^2$$

$$S_2(n) = 2\Re\{E_x(n)E_y(n)\}$$

$$S_2(n) = 2\Im\{E_x(n)E_y^*(n)\}$$





Design rule Stokes equalizer

- Error function to be minimized

$$f(\mathbf{h}) = f(\mathbf{h}_{xx}, \mathbf{h}_{xy}, \mathbf{h}_{yx}, \mathbf{h}_{yy}) =$$
$$= \left(S_{1e}(n) - \hat{S}_1(n)\right)^2 + \left(S_{2e}(n) - \hat{S}_2(n)\right)^2 + \left(S_{3e}(n) - \hat{S}_3(n)\right)^2$$

with:

- $\mathbf{S}_e = [S_{1e}(n), S_{2e}(n), S_{3e}(n)]$ Stokes vector of the equalized signal
- $\hat{\mathbf{S}} = [\hat{S}_1(n), \hat{S}_2(n), \hat{S}_3(n)]$ Stokes vector of the transmitted signal

either known (training sequence) or estimated (decision-directed)

Taps update algorithm – Stokes/CMA

- Rule for adaptively update the equalizer weights:

$$\mathbf{h}_{xx}(n+1) = \mathbf{h}_{xx}(n) - \mu \nabla_{\mathbf{h}_{xx}} f(\mathbf{h}(n))$$
$$\mathbf{h}_{xy}(n+1) = \mathbf{h}_{xy}(n) - \mu \nabla_{\mathbf{h}_{xy}} f(\mathbf{h}(n))$$
$$\mathbf{h}_{yx}(n+1) = \mathbf{h}_{yx}(n) - \mu \nabla_{\mathbf{h}_{xy}} f(\mathbf{h}(n))$$
$$\mathbf{h}_{yy}(n+1) = \mathbf{h}_{yy}(n) - \mu \nabla_{\mathbf{h}_{yy}} f(\mathbf{h}(n))$$

- Evaluation of gradients:

$$\nabla_{\mathbf{h}_{xx}} f(\mathbf{h}(n)) = C_1(n)\mathbf{E}_x^*$$
$$\nabla_{\mathbf{h}_{xy}} f(\mathbf{h}(n)) = C_1(n)\mathbf{E}_y^*$$
$$\nabla_{\mathbf{h}_{yy}} f(\mathbf{h}(n)) = C_2(n)\mathbf{E}_y^*$$
$$\nabla_{\mathbf{h}_{yx}} f(\mathbf{h}(n)) = C_2(n)\mathbf{E}_x^*$$

- **Stokes algorithm**

$$\begin{bmatrix} C_1(n) \\ C_2(n) \end{bmatrix} = \begin{bmatrix} \varepsilon_1(n) & \varepsilon_2(n) \\ \varepsilon_2^*(n) & -\varepsilon_1(n) \end{bmatrix} \begin{bmatrix} E_{xe}(n) \\ E_{ye}(n) \end{bmatrix}$$
$$\varepsilon_1(n) = S_{1e}(n) - \hat{S}_1(n)$$
$$\varepsilon_2(n) = \left(S_{2e}(n) - \hat{S}_2(n)\right) + j\left(S_{3e}(n) - \hat{S}_3(n)\right)$$

- **CMA**

$$\begin{bmatrix} C_1(n) \\ C_2(n) \end{bmatrix} = \begin{bmatrix} \varepsilon_x(n) & 0 \\ 0 & \varepsilon_y(n) \end{bmatrix} \begin{bmatrix} E_{xe}(n) \\ E_{ye}(n) \end{bmatrix}$$
$$\varepsilon_x(n) = \left|E_{xe}(n)\right|^2 - R^2$$
$$\varepsilon_y(n) = \left|E_{ye}(n)\right|^2 - R^2$$

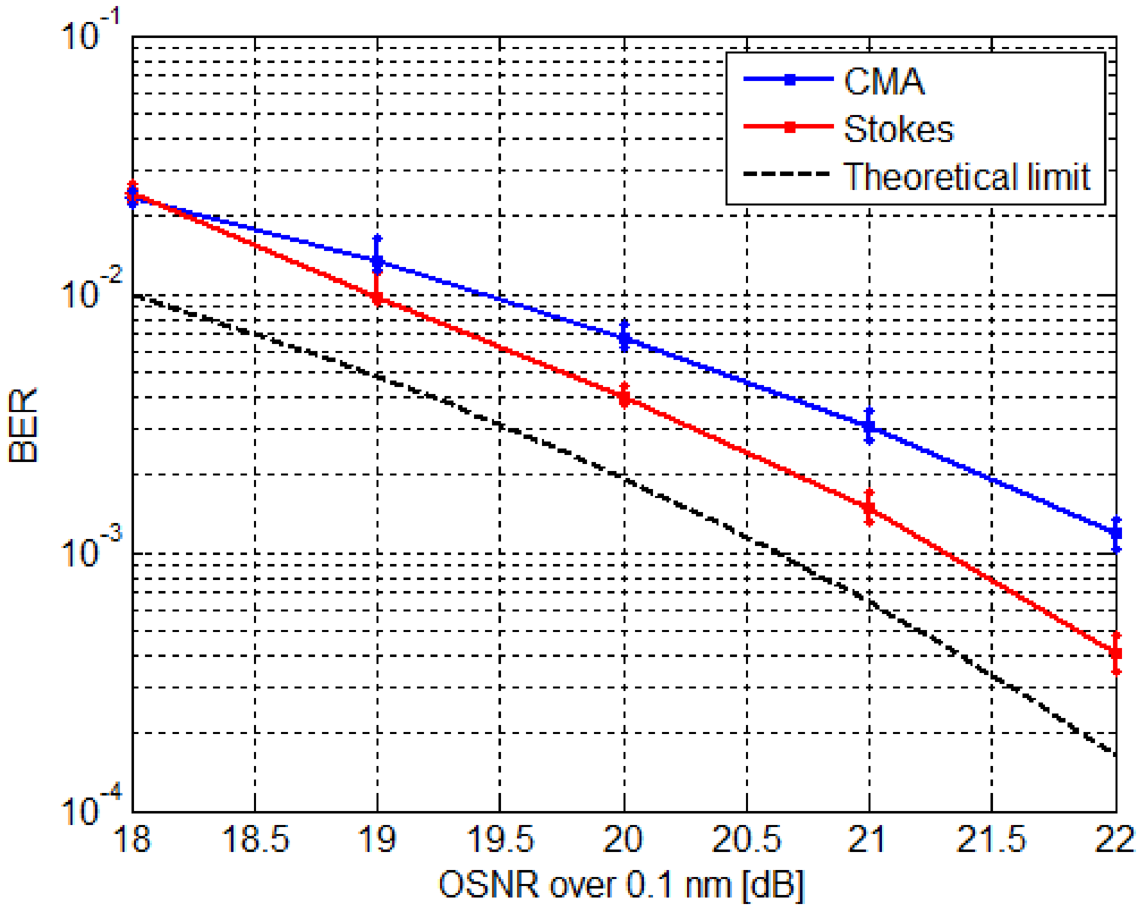
Case study – PM-16QAM

- Symbol rate: R_s=32 Gbaud
- Single-channel
- Nyquist spectrum (raised-cosine with roll-of 0.1)
- Residual CD = 250 ps/nm
- DGD = 1 symbol

- BER values estimated through Monte-Carlo simulation for several combinations of DGD axis and state of polarization (SOP) at the input of the Rx, for a total of ~900 cases

- Equalization using a training sequence, followed by decision-directed operation

BER vs. OSNR



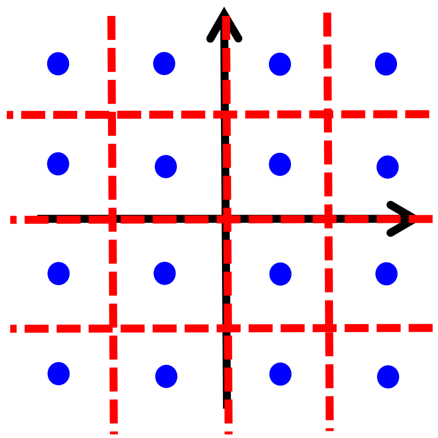
- Number of eq. taps: M = 31.
- Solid lines: average performance
- Vertical bars: range of variations for all considered values of SOP and DGD axis.

- The value of the adaptive equalizer update coefficient μ was optimized for both CMA and Stokes algorithms.

Performance improvement

- Performance can be improved by:

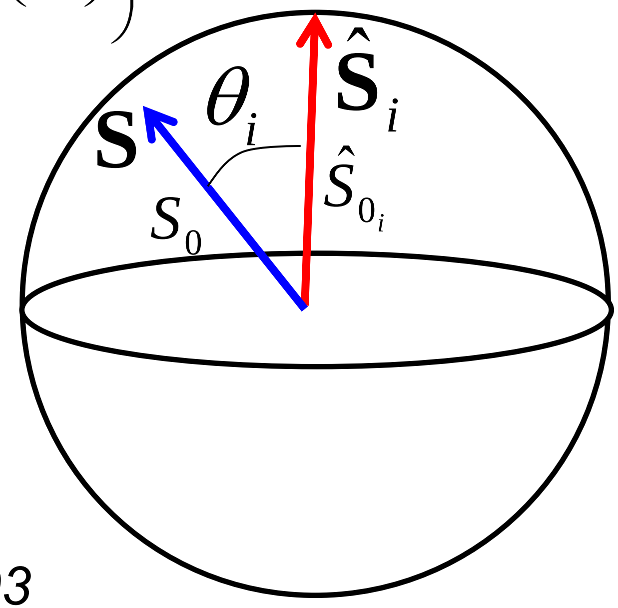
1. Using an adaptive Maximum-Likelihood decision criterion instead of a fixed-threshold one



2. Changing the decision rule in the Stokes space (minimum distance is not optimum)

Statistics of noise in Stokes space

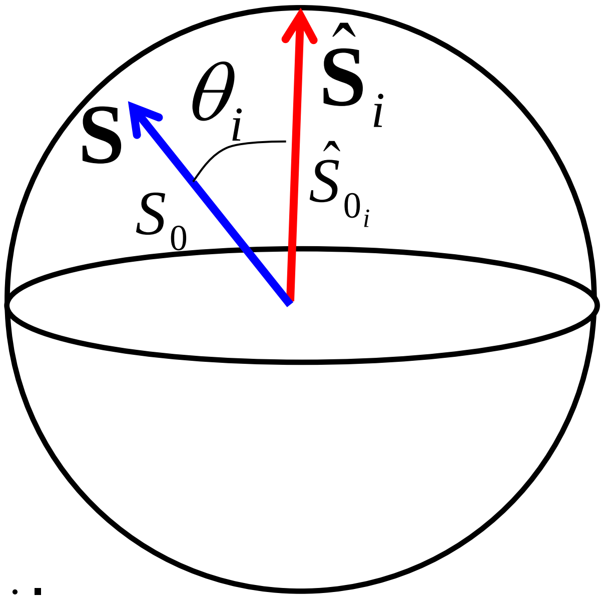
- $\mathbf{S} = (S_1, S_2, S_3)$ = noisy received vector
- $\widehat{\mathbf{S}}_i = (\hat{S}_{1_i}, \hat{S}_{2_i}, \hat{S}_{3_i})$ = ideal un-noisy constellation vector
- PDF of $\mathbf{S} | \widehat{\mathbf{S}}_i$ [*]: $$f_{\mathbf{S}|\widehat{\mathbf{S}}_i} = \frac{e^{-\frac{\hat{S}_{0_i}+S_0}{2\sigma^2}}}{16\pi S_0 \sigma^4} I_0\left(\frac{\sqrt{\hat{S}_{0_i} \cdot S_0}}{\sigma^2} \cos\left(\frac{\theta_i}{2}\right)\right)$$
  - S_0 = magnitude of $\mathbf{S}$
  - $\hat{S}_{0_i}$ = magnitude of $\widehat{\mathbf{S}}_i$
  - θ_i = angle between $\mathbf{S}$ and $\widehat{\mathbf{S}}_i$
  - σ^2 = noise variance in each polarization



[*] P. Poggiolini, "Digital optical transmission systems based on polarization modulation", PhD thesis, 1993

Decision rule

$$f_{\mathbf{s}|\widehat{\mathbf{s}}_i} = \frac{e^{-\frac{\hat{S}_{0_i}+S_0}{2\sigma^2}}}{16\pi S_0 \sigma^4} I_0\left(\frac{\sqrt{\hat{S}_{0_i}\cdot S_0}}{\sigma^2}\cos\left(\frac{\theta_i}{2}\right)\right)$$



- The decision rule can be based on the maximization of $f_{\mathbf{s}|\widehat{\mathbf{s}}_i}$ over all possible noiseless constellation points $\widehat{\mathbf{S}}_i$.
- There are common factors across all possible indices i that can be eliminated → we can apply the ML decision on the formula:

$$p_i = e^{-\frac{S_0}{2\sigma^2}} I_0\left(\frac{\sqrt{\hat{S}_{0_i}\cdot S_0}}{\sigma^2}\cos\left(\frac{\theta_i}{2}\right)\right)$$

Simplified metric in Stokes space

- Taking the logarithm and applying some simplifications, we obtain the following new metric (based on actual statistics in Stokes space):

$$m_i \approx -S_{0_i} + 2\sqrt{\hat{S}_{0_i}}\sqrt{S_0}\cos\left(\frac{\theta_i}{2}\right)$$

NOVEL METRIC

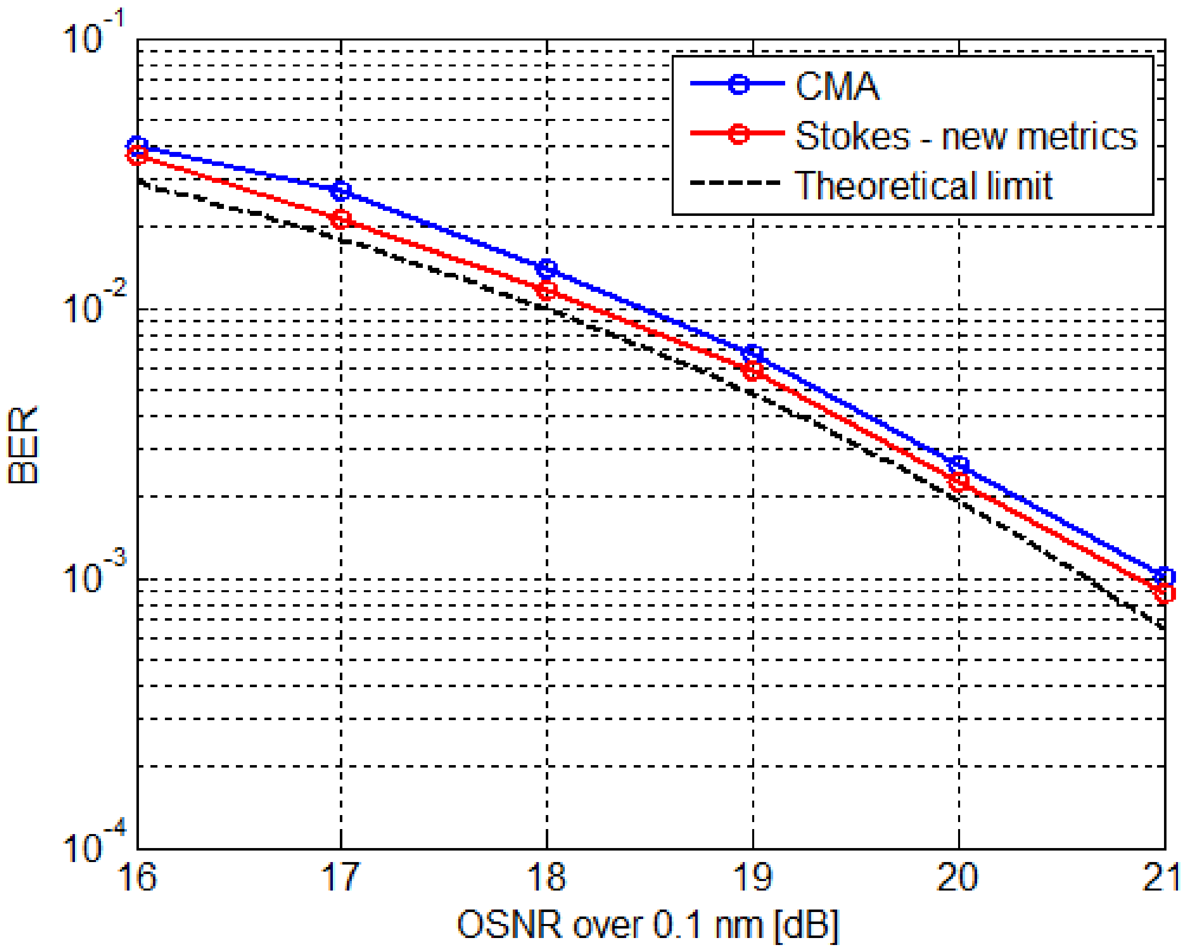
- Minimum-distance metric (based on Gaussian distribution hypothesis)

$$d_i^2 = \left(S_1 - \hat{S}_{1_i}\right)^2 + \left(S_2 - \hat{S}_{2_i}\right)^2 + \left(S_3 - \hat{S}_{3_i}\right)^2 = S_0^2 + \hat{S}_{0_i}^2 - 2\hat{S}_{0_i} S_0 \cos(\theta_i)$$

$$\Longrightarrow -\hat{S}_{0_i}^2 + 2\hat{S}_{0_i} S_0 \cos(\theta_i)$$

MINIMUM DISTANCE METRIC

16QAM with adaptive ML receiver



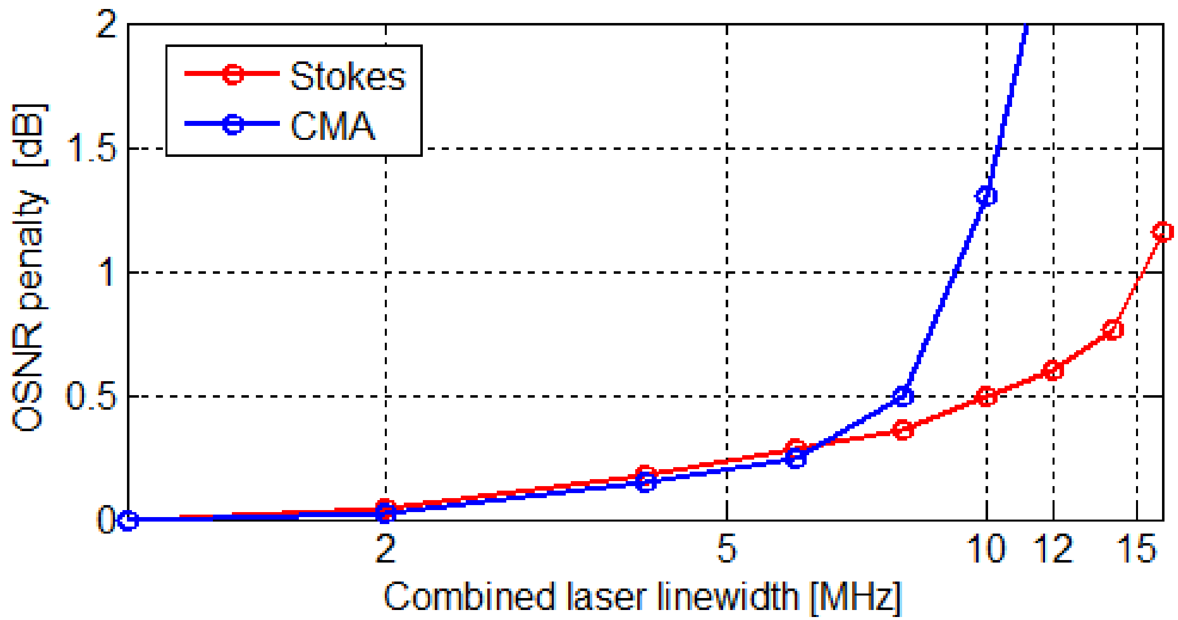
- Number of eq. taps: M = 31.
- Solid lines: average performance (over all considered values of SOP and DGD axis).

- The value of the adaptive equalizer update coefficient μ was optimized for both CMA and Stokes algorithms.

Phase noise tolerance

- CPE block (inserted after the MIMO equalizer), based on the Viterbi&Viterbi algorithm, with QPSK partitioning
- Stokes-space update guarantees that the two polarizations after equalization are perfectly aligned to each other in phase → the phase error estimate can be obtained as an average on both polarizations

- 32 Gbaud PM-16QAM
- Reference BER: $2 \cdot 10^{-2}$



Conclusions

- A novel update algorithm for adaptive MIMO equalizer taps has been proposed and its performance analyzed in a PM-16QAM scenario.
- At the expenses of a slight increase in complexity, it has the following advantages with respect to standard CMA and LMS algorithms:

  <u>With respect to LMS</u>

  - Insensitive to phase noise and frequency offset

  <u>With respect to CMA</u>

  - Not affected by the “degeneracy” problem typical of CMA
  - CPE algorithms can estimate the phase noise by averaging over the two polarizations → nearly doubled phase-noise tolerance

Thank you!

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