# Analytical results on system maximum reach increase through symbol rate optimization

**P. Poggiolini**$^{(1)}$, **Y. Jiang**$^{(1)}$, **A. Carena**$^{(1)}$, **G. Bosco**$^{(1)}$, **F. Forghieri**$^{(2)}$
*(1) DET, Politecnico di Torino, Corso Duca degli Abruzzi, 24, 10129, Torino, Italy. pierluigi.poggiolini@polito.it*
*(2) Cisco Photonics Italy srl, Vimercate (Monza-Brianza), Italy. fforghie@cisco.com*

**Abstract:** We investigate the dependence of maximum system reach on symbol rate. We identify a suitable modeling framework and derive a closed-form formula for the optimum rate, typically 2-to-10 GBaud. Maximum reach gains are between 5% and 20%.

**OCIS codes:** 060.1660 Coherent communications, 060.4510 Optical communications

## 1. Introduction

Recent experiments [1], [2], have shown a rather strong maximum reach gain (20%) in long-haul transmission when a single serial-channel (SC) was broken up into either OFDM subcarriers [1] or FDM quasi-Nyquist subcarriers [2]. Simulative evidence of a dependence of performance on the per-subcarrier symbol rate had also been found in [3]- [5].

In this paper we address this topic with a first objective of finding a theoretical framework that can provide reliable analytical predictions of the optimum symbol rate, as well as of the resulting amount of non-linearity mitigation and reach increase. To this goal, we selected three non-linearity models: the GN model [6], the frequency-domain XPM model [7] and the EGN-model [8]. We then carefully checked their predictions vs. detailed simulations.

Our results indicate that both the GN and the XPM models are inadequate for modeling this phenomenon. The EGN-model is instead very accurate, both in identifying the optimum symbol rate and in predicting the related performance improvement. We also derived from the EGN-model a simple closed-form formula that very reliably predicts the optimum symbol rate for quasi-Nyquist systems with lumped amplification.

As a second objective, we wanted to assess the actual practical potential of symbol rate optimization. The results that we found are in general agreement with [1]- [5]. However, thanks to the modeling results we could explore a wider range of scenarios. We found that the typical optimum symbol rates fall in the range 2-to-10 GBaud. While implementing such low rates with SC transmission is clearly impractical, subcarrier multiplexing over a single carrier by means of DSP-DAC enabled transmitters is certainly feasible, and possibly even advantageous in terms of *reducing* the receiver DSP load [9]. Our results also show that the potential impact of symbol-rate optimization can be substantial in various practical scenarios, with maximum-reach gains ranging between 5% and 20% depending on system data.

## 2. Analytical modeling and simulations

We looked at several test system configurations, having the following *fixed common transmission parameters*: total WDM bandwidth 504 GHz, polarization-multiplexed (PM) QPSK modulation, roll-off 0.05, quasi-Nyquist channel spacing (1.05 times the symbol rate). These parameters imply a fixed total raw bit rate of 1.92 Tb/s, irrespective of the symbol rate per channel, with a raw spectral efficiency of 3.81 b/(s Hz).

We left as free parameter the number of channels $N_{\text{ch}}$ that the overall WDM bandwidth is split into or, equivalently, the per-channel symbol rate $R = 480/N_{\text{ch}}$ (GBaud). As for the link, we looked at either PSCF, SMF, NZDSF, or LS fiber. Attenuation was 0.17, 0.22, 0.22 and 0.22 dB/km, dispersion $D$ was 20.1, 16.7, 3.8 and -1.8 ps/(nm·km), the dual-polarization non-linearity parameter $\gamma$ was 0.8, 1.3, 1.5 and 2.2 $(\text{W}\cdot\text{km})^{-1}$, respectively.

The results for SMF and NZDSF are shown in Fig. 1. The quantity displayed in the pictures, $\tilde{G}_{\text{NLI}}$, is the power-spectral density (PSD) of the non-linear noise NLI falling over the center channel and averaged over it. We also *normalize it* vs. the transmission signal PSD cube, $G_{\text{ch}}^3$. The tilde over $\tilde{G}_{\text{NLI}}$ is a reminder of this normalization. To obtain the power of NLI affecting the center channel, $P_{\text{NLI}}$ , the following de-normalization must be used:

$$P_{\text{NLI}} = \left(\tilde{G}_{\text{NLI}} \cdot R\right) \cdot G_{\text{ch}}^3 \qquad (1)$$

The convenient features of $\tilde{G}_{\text{NLI}}$, are: it is independent of the power per channel launched into the link; it is independent of the symbol rate per channel; a *constant value* of $\tilde{G}_{\text{NLI}}$ across different symbol rates means that the corresponding systems would achieve the *same maximum reach*.

Fig. 1 shows $\tilde{G}_{\rm NLI}$ at 50 spans for SMF (left plot) and at 30 spans for NZDSF (right plot). These span numbers correspond approximately to maximum-reach performance when assuming EDFA amplification with 5.5 dB noise figure. Note though that the plots at other span numbers are qualitatively similar. The markers show the simulation results which were obtained using the technique described in [8]. The curves are model calculations.

Both the GN-model and the XPM-model fail to reproduce the simulated system behavior, whereas the EGN-model confirms its high accuracy, tested in [8], [10]. Specifically, the GN-model curve is essentially flat, that is, it predicts no change of performance vs. the number of channels (or the symbol rate). This behavior can be traced back to the assumption made in the derivation of the GN-model of the signal behaving as Gaussian noise. While it makes the model very simple, it also makes it miss the dip in NLI shown instead by the EGN-model and by the simulations.

The XPM model in its original form [7] does not include single-channel non-linear effects (SCI), so we added the SCI term from [8]. Recently the XPM model, with the addition of some further contributions taken from [8], has been made available as a web resource at the website [11], with the name ‘NLIN’. In Fig. 1 we present both our calculation of XPM+SCI, and the NLIN results obtained from [11]. The figure shows that both these models tend to be accurate for large symbol rates (low channel count) but both depart very substantially from the EGN-model and the simulation results for higher channel count. At the optimum symbol rate, they underestimate $\tilde{G}_{\rm NLI}$ by about 5 dB. The reason for this behavior is their neglect of four-wave-mixing (FWM) effects. FWM becomes prevalent at the optimum symbol rate and this causes the large gap.

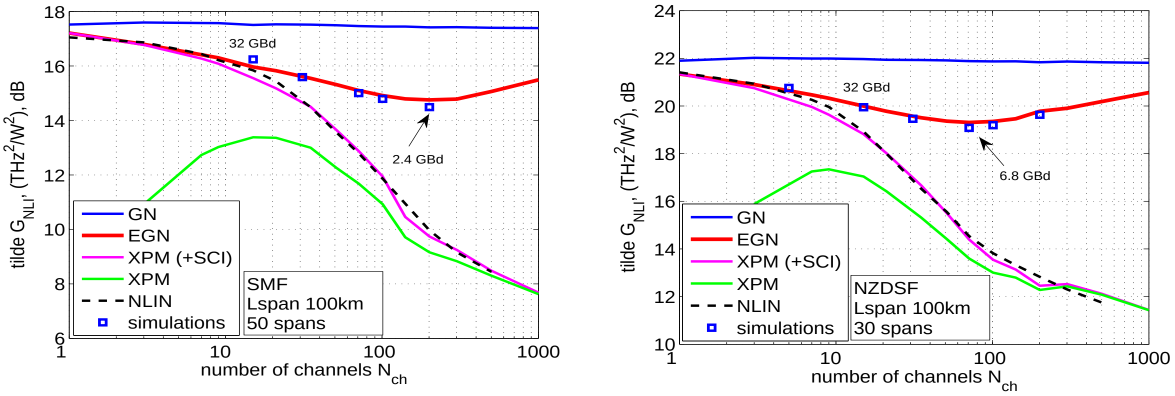

Fig. 1. Normalized average NLI noise power spectral density $\tilde{G}_{\rm NLI}$ over the center channel, vs. the number of channels $N_{\rm ch}$, for a *fixed* WDM bandwidth of 504 GHz. NLI measured at 50 spans of SMF (left) or 30 spans of NZDSF (right). PM-QPSK modulation, quasi-Nyquist: roll-off 0.05, spacing 1.05 times the symbol rate. Solid lines: calculations using the indicated models. Markers: dual-polarization split-step simulations.

In Fig. 1, the $\tilde{G}_{\rm NLI}$ *minimum* for SMF and NZDSF is located at about 200 and 70 channels, i.e., at about 2.4 and 6.8 GBaud, respectively. The NLI mitigation vs. the current industry-standard 32 GBaud (15 channels in the plots) is 1.2 and 0.7 dB, respectively. This NLI mitigation does not directly translate into max-reach gains, though. The approximate value of max-reach increase is [6]: $\Delta L^{\rm max}_{\rm dB} \approx -\frac{1}{3}\Delta\tilde{G}_{\rm NLI,dB}$, where $L^{\rm max}$ is the max-reach and $\Delta$ represents the ratio of two values of the same quantity. As a result, the 1.2 dB and 0.7 dB mitigation of the normalized NLI PSD $\tilde{G}_{\rm NLI}$ leads to 0.4 dB and 0.25 dB (or 10% and 6%) max-reach increases for SMF and NZDSF, respectively.

To double check this prediction, we ran detailed max-reach simulations for the same system described above, at different symbol rates. NZDSF results were available at the time of submission of this paper. In Fig. 2 (left) for NZDSF we show the EGN-model reach predictions as lines and the simulation results (based on direct error counting at the receiver) as markers. The correspondence is good, confirming the expected 6% max-reach increase at the optimum rate predicted through the EGN-model, vs. the 32 GBaud scenario.

These results are in general good agreement with the simulative results that had appeared in [3], [4]. The experimental results of [2] predicted a larger max-reach gain (20%). However that was found assuming a WDM bandwidth of only 25 GHz (vs. 504 GHz as in Figs. 1-2). We ran both EGN calculations and simulations of a system similar to [2] and also found a 20% max reach increase, which provides evidence that NLI mitigation decreases as the WDM bandwidth goes up. On the other hand, it appears that NLI mitigation increases substantially when shortening the span. For instance, the EGN-model calculations predict that the mitigation (between 32 GBaud and $R_{\rm opt}$) is 1.8 dB over SMF with $L_{\rm span}$ = 50 km, and 2 dB over PSCF with $L_{\rm span}$ = 60 km. Finally, not shown for lack of space, we redid the same plots as in Fig. 1 for PM-16QAM. The curves are qualitatively similar and show the exact same $R_{\rm opt}$. The gain is however uniformly reduced. For SMF 100km spans, it is 0.7 dB rather than 1.2, between 32 GBaud and $R_{\rm opt}$.

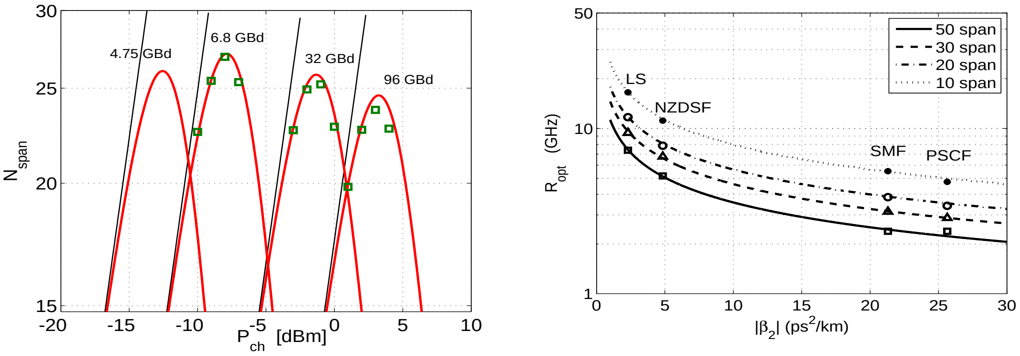

Fig. 2. Left plot: reach curves at BER=$4 \cdot 10^{-3}$ over NZDSF, $L_{\text{span}}$=100 km. PM-QPSK, roll-off 0.5, spacing 1.05·$R$. EDFA noise figure 6 dB. Solid lines: EGN-model predictions. Markers: simulation results available at time of submission. Right plot: value of the optimum symbol rate as predicted by Eq. 2 (solid lines) or by full EGN-model (markers), as a function of fiber dispersion and of link length in number of spans ($L_{\text{span}}$=100 km).

## 3. Closed-form optimum symbol rate formula

We derived a closed-form expression of the optimum symbol rate, by using results from the derivation of the asymptotic closed-form EGN model proposed in [10]. For quasi-Nyquist systems like the ones considered in this paper, with all identical spans, the optimum rate is:

$$R_{\text{opt}} = \sqrt{2/(\pi\,|\,\beta_2\,|\,L_{\text{span}}\,N_{\text{span}})} \tag{2}$$

We show in Fig. 2 an extensive test of Eq. (2). The markers are drawn by finding the optimum symbol rates from complete EGN-model $\tilde{G}_{\text{NLI}}$ curves like those of Fig. 1, for the same PM-QPSK transmission used in Fig. 1. The solid lines are Eq. (2). The plot shows an excellent match, vs. the wide range of dispersions and number of spans addressed. In addition, we tested a few data points for $L_{\text{span}} = 50$ and 60 km, over SMF and PSCF, and for 25 GHz total system bandwidth over SMF, similar to [2]. In these cases too Eq. (2) proved accurate. The formula indicates that the optimum rate is a function not only of the accumulated dispersion per span $|\,\beta_2\,| \cdot L_{\text{span}}$ but also of the link length through $N_{\text{span}}$. This actually agrees with a simulative indication from [4]. Owing to the square root in Eq. (2), the range of optimum rates is relatively narrow. It goes above 10 GBaud only for LS fibers and for relatively short NZDSF links.

## 4. Comments and conclusion

We have identified the EGN-model as a suitable tool for studying system performance optimization vs. symbol rate. The EGN-model analytical and simulative results also agree well with prior simulative and experimental papers [1]-[5] and provide a consistent framework to reliably extend the study to a variety of scenarios. Our initial assessment shows that in typical EDFA terrestrial systems with 100km spans the NLI mitigation from current 32 GBaud to the optimum rate (which is in the range 2-to-10 GBaud) allows on the order of 6%-10% max reach increase for PM-QPSK. The effect is however less pronounced for higher-order constellations.

Nonetheless, we believe these results may significantly influence future trends. In particular, the general industry push towards higher symbol rates must be weighed vs. the greater penalties that are incurred there. In Fig. 1, the NLI gap between $R_{\text{opt}}$=2.4 and 32 GBaud is 1.2 dB, but it goes up to 2 dB between $R_{\text{opt}}$ and 100 GBaud. It is a full 3 dB between $R_{\text{opt}}$=2.8 GBaud and 100 GBaud over 60-km-spans PSCF. This means that, apart from implementation penalties, there may be a fundamental disadvantage for straight serial-rate increase. While the trend for increasing the per-carrier throughput is likely to continue, it may have to evolve towards electronic subcarrier multiplexing to avoid a quite substantial maximum reach penalty.

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