Digital Nonlinear Equalization
for Long-Haul Optical Transmission Systems

Fernando P. Guiomar

Supervisor: Prof. Armando Humberto Moreira Nolasco Pinto

Department of Electronics, Telecommunications and Informatics, University of Aveiro, Aveiro, Portugal
Instituto de Telecomunicações - Pólo de Aveiro, Portugal
Who am I?

- So many names!
  - **Fernando** Pedro Pereira **Guiomar**
Who am I?

- So many names!
  - **Fernando** Pedro Pereira **Guiomar**
- I did my PhD on **Digital Nonlinear Equalization for Optical Transmission Systems**
  - Instituto de Telecomunicações and University of Aveiro, Portugal.
Who am I?

- So many names!
  - **Fernando** Pedro Pereira **Guiomar**

- I did my PhD on **Digital Nonlinear Equalization for Optical Transmission Systems**
  - Instituto de Telecomunicações and University of Aveiro, Portugal.

- Other things I like to do:
  - Football, music, cinema, travelling...
Outline

- Summary of my PhD Thesis Work;
  - Review of Digital Backpropagation;
  - Frequency Domain Volterra Series Nonlinear Equalizer;
  - Time Domain Volterra Series Nonlinear Equalizer;
  - Multi-Carrier Digital Backpropagation.

- Some Open Topics on Nonlinear Equalization;

- The FLEX-ON Project;
  - Main Scientific Objectives;
  - Dissemination and Public Engagement.
Review of Digital Backpropagation
Digital Backpropagation - Operation Principle

- Effective linear+nonlinear compensation requires channel inversion techniques;

- Signal propagation in the direct fiber direction can be described by:

\[
\frac{\partial A_{x/y}}{\partial z} = -\frac{\alpha}{2} A_{x/y} - i\frac{\beta_2}{2} \frac{\partial^2 A_{x/y}}{\partial t^2} + i\frac{8}{9}\gamma \left(|A_x|^2 + |A_y|^2\right) A_{x/y},
\]
Digital Backpropagation - Operation Principle

- Effective linear+nonlinear compensation requires **channel inversion** techniques;
  - Signal propagation in the **reverse fiber direction** can be described by:

\[
-\frac{\partial A_x/y}{\partial z} = +\frac{\alpha}{2}A_x/y + i\frac{\beta_2}{2}\frac{\partial^2 A_x/y}{\partial^2 t} - i\frac{8}{9}\gamma (|A_x|^2 + |A_y|^2) A_x/y,
\]
Effective linear+nonlinear compensation requires channel inversion techniques;

Signal propagation in the reverse fiber direction can be described by:

\[-\frac{\partial A_{x/y}}{\partial z} = +\frac{\alpha}{2} A_{x/y} + i \frac{\beta_2}{2} \frac{\partial^2 A_{x/y}}{\partial^2 t} - i \frac{8}{9} \gamma (|A_x|^2 + |A_y|^2) A_{x/y},\]

This corresponds to propagate the received signal through a virtual fiber with parameters with the opposite sign \((-\alpha, -\beta_2 e^{-\gamma}).\)
Effective linear+nonlinear compensation requires channel inversion techniques;

- Signal propagation in the reverse fiber direction can be described by:

\[-\frac{\partial A_{x/y}}{\partial z} = + \frac{\alpha}{2} A_{x/y} + i \frac{\beta_2}{2} \frac{\partial^2 A_{x/y}}{\partial t^2} - i \frac{8}{9} \gamma \left( |A_x|^2 + |A_y|^2 \right) A_{x/y},\]

- This corresponds to propagate the received signal through a virtual fiber with parameters with the opposite sign \((-\alpha, -\beta_2 e^{-\gamma}).\)

- This method can be digitally applied at the receiver-side for universal impairment compensation;

Digital Backpropagation - Operation Principle

- Effective linear+nonlinear compensation requires channel inversion techniques;
  - Signal propagation in the reverse fiber direction can be described by:
    \[
    -\frac{\partial A_{x/y}}{\partial z} = +\frac{\alpha}{2} A_{x/y} + i\frac{\beta_2}{2} \frac{\partial^2 A_{x/y}}{\partial^2 t} - i\frac{8}{9} \gamma (|A_x|^2 + |A_y|^2) A_{x/y},
    \]
  - This corresponds to propagate the received signal through a virtual fiber with parameters with the opposite sign \((-\alpha, -\beta_2 e^{-\gamma})\).

- This method can be digitally applied at the receiver-side for universal impairment compensation;
- Equalization performance and computational efficiency strongly depend on the numerical method used to apply DBP.

• BP-SSFM has been the most widely used numerical method to solve DBP;

BP-SSFM has been the most widely used numerical method to solve DBP;

\[
\begin{align*}
- j \gamma h_{eff} \xi \\
| \cdot |^2 + \times \quad \text{exp(\cdot)} \\
\times \quad \text{FFT} \\
\text{exp} \left( \frac{i \omega^2 h}{2} \right) \\
\text{IFFT} \\
A_{eqx}[1] \\
A_{eqx}[2] \\
A_{eqx}[N_{FFT}] \\
\times N_{steps} \\
\end{align*}
\]
BP-SSFM has been the most widely used numerical method to solve DBP;

If spatial and temporal resolutions are sufficiently high, BP-SSFM enables to **fully** compensate all the deterministic impairments.
Inverse Volterra Series Transfer Function

3.1) Analytical Description
**DBP alternative:** third-order Volterra series expansion of the inverse NLSE in frequency-domain:

\[
\tilde{A}_{x/y}(\omega, z-L_s) \approx K_1(\omega, L_s)\tilde{A}_{x/y}(\omega, z) + \Gamma(\omega, L_s)\iint K_3(\omega, \omega_j, \omega_k) \tilde{P}(\omega_j, \omega_k, z)\tilde{A}_{x/y}(\omega + \omega_j - \omega_k, z) \partial\omega_j\partial\omega_k,
\]

- Linear term
- 3rd-order nonlinear term

DBP alternative: third-order Volterra series expansion of the inverse NLSE in frequency-domain:

\[
\tilde{A}_{x/y}(\omega, z-L_s) \approx K_1(\omega, L_s)\tilde{A}_{x/y}(\omega, z) + \Gamma(\omega, L_s) \iint K_3(\omega, \omega_j, \omega_k) \tilde{P}(\omega_j, \omega_k, z) \tilde{A}_{x/y}(\omega + \omega_j - \omega_k, z) \partial\omega_j \partial\omega_k,
\]

where \(\tilde{P}(\omega_j, \omega_k, z)\) includes the inter-polarization crosstalk,

\[
\tilde{P}(\omega_j, \omega_k, z) = \tilde{A}_x(\omega_k, z)\tilde{A}_x^*(\omega_j, z) + \tilde{A}_y(\omega_k, z)\tilde{A}_y^*(\omega_j, z),
\]
**DBP alternative:** third-order Volterra series expansion of the inverse NLSE in frequency-domain:

\[
\tilde{A}_{x/y}(\omega, z - L_s) \approx K_1(\omega, L_s) \tilde{A}_{x/y}(\omega, z) + \Gamma(\omega, L_s) \int \int K_3(\omega, \omega_j, \omega_k) \tilde{P}(\omega_j, \omega_k, z) \tilde{A}_{x/y}(\omega + \omega_j - \omega_k, z) \partial \omega_j \partial \omega_k,
\]

where \( \tilde{P}(\omega_j, \omega_k, z) \) includes the inter-polarization crosstalk,

\[
\tilde{P}(\omega_j, \omega_k, z) = \tilde{A}_x(\omega_k, z) \tilde{A}_x^*(\omega_j, z) + \tilde{A}_y(\omega_k, z) \tilde{A}_y^*(\omega_j, z),
\]

\( K_1(\omega, z) \) is the inverse linear kernel,

\[
K_1(\omega, z) = \exp \left( \frac{\alpha}{2} L_s - i \frac{\beta_2}{2} \omega^2 z \right),
\]
● **DBP alternative**: third-order Volterra series expansion of the inverse NLSE in frequency-domain:

\[
\tilde{A}_{x/y}(\omega, z-L_s) \approx K_1(\omega, L_s) \tilde{A}_{x/y}(\omega, z) + \Gamma(\omega, L_s) \int K_3(\omega, \omega_j, \omega_k) \tilde{P}(\omega_j, \omega_k, z) \tilde{A}_{x/y}(\omega + \omega_j - \omega_k, z) \partial \omega_j \partial \omega_k,
\]

where \( \tilde{P}(\omega_j, \omega_k, z) \) includes the inter-polarization crosstalk,

\[
\tilde{P}(\omega_j, \omega_k, z) = \tilde{A}_x(\omega_k, z) \tilde{A}_x^*(\omega_j, z) + \tilde{A}_y(\omega_k, z) \tilde{A}_y^*(\omega_j, z),
\]

\( K_1(\omega, z) \) is the inverse linear kernel,

\[
K_1(\omega, z) = \exp\left(\frac{\alpha}{2} L_s - i \frac{\beta_2}{2} \omega^2 z\right),
\]

\( K_3(\omega, \omega_j, \omega_k) \) is the inverse nonlinear kernel,

\[
K_3(\omega, \omega_j, \omega_k) = \frac{1 - \exp\left(\alpha L_s - i \beta_2 (\omega_k - \omega)(\omega_k - \omega_j) L_s\right)}{-\alpha + i \beta_2 (\omega_k - \omega)(\omega_k - \omega_j)},
\]

and \( \Gamma(\omega, z) \) is a one-dimensional frequency-dependent nonlinear term,

\[
\Gamma(\omega, z) = -i \xi \frac{8}{9} \gamma K_1(\omega, z).
\]
The Volterra series nonlinear equalizer in frequency domain is based on an entry-wise product of $N \times N$ matrices:

$$\tilde{N}_{x/y}(\omega_n, z) = K_3(\omega_n) \circ \tilde{P}(z) \circ \tilde{A}_{x/y}(\omega_n, z)$$

Highly parallel implementation, but with $O(N^2)$ complexity per processed sample.
3.2) Experimental Results
VSNE - Experimental Setup

- Experimental setup of a **100G PM-QPSK experiment**, carried out in collaboration with Politecnico di Torino, Italy, in the framework of the EURO-FOS project:

- Main features:
  - Single-channel PM-QPSK @ 120 Gb/s;
  - Optical Nyquist pulse shaping;
  - Propagation over **NZDSF** spans with 100 km each;
  - Sampling @ 50 Gsa/s - ~1.6 samples per symbol.

---

Single-channel 120 Gb/s PM-QPSK transmission over an NZDSF link:

- After 1600 km
- After 3100 km

Single-channel 120 Gb/s PM-QPSK transmission over an NZDSF link:

After 1600 km

- The single-polarization DBP-SSFM and DBP-VSTF increase the nonlinear tolerance by \( \sim 1 \) dB;

After 3100 km

- The single-polarization DBP-SSFM and DBP-VSTF increase the nonlinear tolerance by \( \sim 1 \) dB;

Single-channel 120 Gb/s PM-QPSK transmission over an NZDSF link:

- After 1600 km:
  - $\times 1.7$ dB
  - $\times 0.7$ dB

- After 3100 km:
  - $\times 1.6$ dB
  - $\times 0.7$ dB

The single-polarization DBP-SSFM and DBP-VSTF increase the nonlinear tolerance by $\sim 1$ dB;

- Another 0.7 dB is obtained by the dual-polarization models, with negligible added complexity;

- The DBP-SSFM and DBP-VSTF are shown to yield similar accuracy.

Single-channel 120 Gb/s PM-QPSK transmission over an NZDSF link:

- After 1600 km
  - 2 × 5 × 2.5 × 1 dB
  - 1.7 dB
  - 0.7 dB

- After 3100 km
  - 1.6 × 3.5 × 2.2 × 0.9 dB
  - 1.6 dB
  - 0.9 dB

- The single-polarization DBP-SSFM and DBP-VSTF increase the nonlinear tolerance by ∼1 dB;
- Another 0.7 dB is obtained by the dual-polarization models, with negligible added complexity;
- The DBP-SSFM and DBP-VSTF are shown to yield similar accuracy.
- But, can we reduce the complexity of the DBP-VSTF?

Frequency Domain VSNE

4.1) Analytical Description
VSNE - 3rd Order Kernel

NZDSF: \( N = 32; n = 16 \)

SSMF:

- Interesting **symmetries** can be exploited to avoid replication of operations;
  - Some of the symmetries are also shared with the \( \tilde{A}_{x/y} \) matrices.

- The \( K_3 \) pattern strongly depends on the accumulated dispersion;
- The real part of \( K_3 \) is maximum at the iXPM+iSPM contribution.

\[
\begin{align*}
\text{iSPM} & \rightarrow j = k = n; \\
\text{iXPM} & \rightarrow j = k \neq n; \\
\text{iFWM} & \rightarrow \text{otherwise}
\end{align*}
\]
Starting from the iXPM contribution (main diagonal + \( n \)-th column), the \( K_3 \) kernel can be reconstructed from its symmetric column/diagonal pairs:

\[
\begin{align*}
|\text{Re}[K_3]| & \quad \quad \quad N_K = 0 & \quad \quad \quad |\text{Im}[K_3]| \\
\end{align*}
\]

The full VSNE kernel is then decomposed into a set of \( N_K \) parallel frequency-domain filters;

- **symVSNE**: complexity is reduced from \( O(N^2) \) to \( O(N_kN) \);
- **simVSNE**: constant-coefficient approximation, numerical complexity is \( O(N_k) \).
Backpropagation using VSNE-based Equalizers

- DBP implementation using CDE and VSNE for linear and nonlinear compensation:

\[ A_{in} \rightarrow \text{CDE} \rightarrow A_{eq} \]

\[ A_{in} \rightarrow \text{CDE} \rightarrow A_{eq} \]
Backpropagation using VSNE-based Equalizers

- DBP implementation using CDE and VSNE for linear and nonlinear compensation:

- The VSNE module is applied in parallel with CDE;
  - The performance is improved by perturbing the CDE output with a nonlinear error signal.

Backpropagation using VSNE-based Equalizers

- DBP implementation using CDE and VSNE for linear and nonlinear compensation:

  - The VSNE module is applied in parallel with CDE;
    - The performance is improved by perturbing the CDE output with a nonlinear error signal.
  - The VSNE can be subdivided into a set of $N_k$ parallel simVSNE equalizers.
    - Nonlinear equalization becomes an highly granular add-on;
    - Easier trade-off between performance and complexity, depending on the DSP resources.
Frequency Domain VSNE

4.2) Simulation and Experimental Results
Simulation results for a $20 \times 80$ km $224$ Gb/s PM-16QAM transmission system:

- **Number of CMs per sample:**

<table>
<thead>
<tr>
<th>$N_k$</th>
<th>$N = 32$</th>
<th>$N = 128$</th>
<th>$N = 32$</th>
<th>$N = 128$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>6</td>
<td>6</td>
<td>6</td>
<td>6</td>
</tr>
<tr>
<td>2</td>
<td>134.9</td>
<td>519</td>
<td>17.6</td>
<td>17.9</td>
</tr>
<tr>
<td>3</td>
<td>247.6</td>
<td>1015.2</td>
<td>28.9</td>
<td>29.7</td>
</tr>
<tr>
<td>4</td>
<td>346.3</td>
<td>1495.8</td>
<td>39.7</td>
<td>41.4</td>
</tr>
</tbody>
</table>

Simulation results for a $20 \times 80$ km $224$ Gb/s PM-16QAM transmission system:

- The $\text{simVSNE}$ error is negligible for low-dispersion fibers;

### Number of CMs per sample:

<table>
<thead>
<tr>
<th>$N_k$</th>
<th>$N = 32$</th>
<th>$N = 128$</th>
<th>$N = 32$</th>
<th>$N = 128$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>6</td>
<td>6</td>
<td>6</td>
<td>6</td>
</tr>
<tr>
<td>2</td>
<td>134.9</td>
<td>519</td>
<td>17.6</td>
<td>17.9</td>
</tr>
<tr>
<td>3</td>
<td>247.6</td>
<td>1015.2</td>
<td>28.9</td>
<td>29.7</td>
</tr>
<tr>
<td>4</td>
<td>346.3</td>
<td>1495.8</td>
<td>39.7</td>
<td>41.4</td>
</tr>
</tbody>
</table>
Simulation results for a $20 \times 80 \text{ km} \ 224 \text{ Gb/s PM-16QAM transmission system:}$

<table>
<thead>
<tr>
<th>$N_k$</th>
<th>$N = 32$</th>
<th>$N = 128$</th>
<th>$N = 32$</th>
<th>$N = 128$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>6</td>
<td>6</td>
<td>6</td>
<td>6</td>
</tr>
<tr>
<td>2</td>
<td>134.9</td>
<td>519</td>
<td>17.6</td>
<td>17.9</td>
</tr>
<tr>
<td>3</td>
<td>247.6</td>
<td>1015.2</td>
<td>28.9</td>
<td>29.7</td>
</tr>
<tr>
<td>4</td>
<td>346.3</td>
<td>1495.8</td>
<td>39.7</td>
<td>41.4</td>
</tr>
</tbody>
</table>

- The simVSNE error is negligible for low-dispersion fibers;
- With large $\beta_2$, the iXPM-like regions in the VSNE kernel tend to become narrower;
- Still, for small $N_k$, the constant-coefficient assumption remains valid!

• **100G PM-64QAM transmission system** implemented in collaboration Politecnico di Torino and Istituto Superiore Mario Boella, Italy:
  
  • 124.8 Gb/s PM-64QAM signal;
  • Recirculating loop composed of $2 \times 54.44$ km of PSCF (150 $\mu$m$^2$); 
  • 10 WDM channels spaced by 50 GHz;
  • Digital Nyquist pulse shaping.

---

• BER performance of 124.8 Gb/s PM-64QAM after linear and nonlinear equalization:

![Graph showing BER and maximum reach versus input power and number of spans.](image-url)
• BER performance of 124.8 Gb/s PM-64QAM after linear and nonlinear equalization:

F. P. Guiomar et al., “Transmission of PM-64QAM over 1524 km of PSCF using Fully-Blind Equalization and Volterra-Based Nonlinear Mitigation”, in Proc. ECOC, paper We.3.3.3, Cannes, September, 2014.
simVSNE - Experimental Demonstration

- BER performance of 124.8 Gb/s PM-64QAM after linear and nonlinear equalization:

- The simVSNE is able to match the BP-SSF maximum performance (1 span per step);
- Maximum reach is increased from 1198 km to 1524 km (27% increase);
- Optimum power in increased from -7 dBm to -5 dBm (2 dB increase in nonlinear tolerance).

F. P. Guiomar et al, “Transmission of PM-64QAM over 1524 km of PSCF using Fully-Blind Equalization and Volterra-Based Nonlinear Mitigation”, in Proc. ECOC, paper We.3.3.3, Cannes, September, 2014.
• Computational effort comparison between BP-SSF, symVSNE and simVSNE:

<table>
<thead>
<tr>
<th>$N_{\text{steps}}$</th>
<th>$N$</th>
<th>SSF</th>
<th>symVSNE</th>
<th>simVSNE</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>$N_{\text{CMS}}$</td>
<td>$N_k$</td>
<td>$N_{\text{CMS}}$</td>
</tr>
<tr>
<td>28</td>
<td>64</td>
<td>523</td>
<td>4</td>
<td>6113</td>
</tr>
<tr>
<td>14</td>
<td>64</td>
<td>266</td>
<td>4</td>
<td>3063</td>
</tr>
<tr>
<td>7</td>
<td>64</td>
<td>--</td>
<td>5</td>
<td>1794</td>
</tr>
<tr>
<td>4</td>
<td>128</td>
<td>--</td>
<td>8</td>
<td>3316</td>
</tr>
<tr>
<td>3</td>
<td>128</td>
<td>--</td>
<td>9</td>
<td>2709</td>
</tr>
<tr>
<td>2</td>
<td>256</td>
<td>--</td>
<td>15</td>
<td>6164</td>
</tr>
<tr>
<td>1</td>
<td>256</td>
<td>--</td>
<td>19</td>
<td>3658</td>
</tr>
</tbody>
</table>

• The SSF method requires a minimum of **14 steps**, yielding **266 CMs**;
### Computational Effort Comparison

#### Computational effort comparison between BP-SSF, symVSNE and simVSNE:

<table>
<thead>
<tr>
<th>( N_{\text{steps}} )</th>
<th>( N )</th>
<th>SSF</th>
<th>symVSNE</th>
<th>simVSNE</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( N_{\text{CMS}} )</td>
<td>( N_k )</td>
<td>( N_{\text{CMS}} )</td>
<td>( N_k )</td>
</tr>
<tr>
<td>28</td>
<td>64</td>
<td>523</td>
<td>4</td>
<td>6113</td>
</tr>
<tr>
<td>14</td>
<td>64</td>
<td>266</td>
<td>4</td>
<td>3063</td>
</tr>
<tr>
<td>7</td>
<td>64</td>
<td>--</td>
<td>5</td>
<td>1794</td>
</tr>
<tr>
<td>4</td>
<td>128</td>
<td>--</td>
<td>8</td>
<td>3316</td>
</tr>
<tr>
<td>3</td>
<td>128</td>
<td>--</td>
<td>9</td>
<td>2709</td>
</tr>
<tr>
<td>2</td>
<td>256</td>
<td>--</td>
<td>15</td>
<td>6164</td>
</tr>
<tr>
<td>1</td>
<td>256</td>
<td>--</td>
<td>19</td>
<td>3658</td>
</tr>
</tbody>
</table>

- The SSF method requires a minimum of **14 steps**, yielding **266 CMs**;
- The symVSNE can be applied with only **1 step**, but incurs higher complexity;
### Computational Effort Comparison

<table>
<thead>
<tr>
<th>$N_{\text{steps}}$</th>
<th>$N$</th>
<th>SSF</th>
<th>symVSNE</th>
<th>simVSNE</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>$N_{\text{CMS}}$</td>
<td>$N_k$</td>
<td>$N_{\text{CMS}}$</td>
</tr>
<tr>
<td>28</td>
<td>64</td>
<td>523</td>
<td>4</td>
<td>6113</td>
</tr>
<tr>
<td>14</td>
<td>64</td>
<td>266</td>
<td>4</td>
<td>3063</td>
</tr>
<tr>
<td>7</td>
<td>64</td>
<td>--</td>
<td>5</td>
<td>1794</td>
</tr>
<tr>
<td>4</td>
<td>128</td>
<td>--</td>
<td>8</td>
<td>3316</td>
</tr>
<tr>
<td>3</td>
<td>128</td>
<td>--</td>
<td>9</td>
<td>2709</td>
</tr>
<tr>
<td>2</td>
<td>256</td>
<td>--</td>
<td>15</td>
<td>6164</td>
</tr>
<tr>
<td>1</td>
<td>256</td>
<td>--</td>
<td>19</td>
<td>3658</td>
</tr>
</tbody>
</table>

- The SSF method requires a minimum of **14 steps**, yielding **266 CMs**;
- The symVSNE can be applied with only **1 step**, but incurs higher complexity;
- The simVSNE provides an optimized balance between latency and complexity: *4 steps* and **213 CMs**.

---

Time Domain VSNE

5.1) Analytical Description
• Frequency-domain iXPM filter - \text{simVSNE}[0]:

\[ \tilde{A}_x^{\text{iXPM}}(\omega_n) = \kappa \left[ \tilde{A}_y^{\text{CD}}(\omega_n) \left( 2 \sum_{k=1}^{N} |\tilde{A}_{x/y}(\omega_k)|^2 + \sum_{k=1}^{N} |\tilde{A}_{y/x}(\omega_k)|^2 - \tilde{\chi}(\omega_n) \right) + \tilde{A}_y^{\text{CD}}(\omega_n) \sum_{k=1}^{N} \tilde{A}_{x/y}(\omega_k) \tilde{A}_{y/x}^*(\omega_k) \right], \]
• Frequency-domain iXPM filter - simVSNE[0]:

\[
\tilde{A}_{x/y}(\omega_n) = \kappa \left[ \tilde{A}_{CD}^{x/y}(\omega_n) \left( 2 \sum_{k=1}^{N} |\tilde{A}_{x/y}(\omega_k)|^2 + \sum_{k=1}^{N} |\tilde{A}_{y/x}(\omega_k)|^2 - \tilde{\chi}(\omega_n) \right) + \tilde{A}_{CD}^{x/y}(\omega_n) \sum_{k=1}^{N} \tilde{A}_{x/y}(\omega_k) \tilde{A}_{y/x}^*(\omega_k) \right],
\]

• Applying an inverse Fourier transform, we obtain a time-domain equivalent:

\[
A_{x/y}(t_n) = \kappa \left[ A_{CD}^{x/y}(t_n) \left( 2P_{x/y}(t_n) + P_{y/x}(t_n) \right) + A_{y/x}^{x/y}(t_n)P_{xy/yx}(t_n) - \chi_{x/y}(t_n) \right],
\]

\[
P_{x/y}(t_n) = \frac{1}{N_{\text{NLE}}} \sum_{k \in K} |A_{x/y}(t_k)|^2, \quad P_{xy}(t_n) = \frac{1}{N_{\text{NLE}}} \sum_{k \in K} A_x(t_k)A_y^*(t_k), \quad P_{yx}(t_n) = P_{xy}^*(t_n),
\]
**Time Domain VSNE - Analytical Formulation**

- **Frequency-domain iXPM filter - simVSNE[0]:**

\[
\tilde{A}_{x/y}^{\text{iXPM}}(\omega_n) = \kappa \left[ \tilde{A}_{x/y}^{\text{CD}}(\omega_n) \left( 2 \sum_{k=1}^{N} |\tilde{A}_{x/y}(\omega_k)|^2 + \sum_{k=1}^{N} |\tilde{A}_{y/x}(\omega_k)|^2 - \tilde{\chi}(\omega_n) \right) + \tilde{A}_{y/x}^{\text{CD}}(\omega_n) \sum_{k=1}^{N} \tilde{A}_{x/y}(\omega_k) \tilde{A}_{y/x}^{*}(\omega_k) \right],
\]

- **Applying an inverse Fourier transform, we obtain a time-domain equivalent:**

\[
A_{x/y}^{\text{iXPM}}(t_n) = \kappa \left[ A_{x/y}^{\text{CD}}(t_n) \left( 2P_{x/y}(t_n) + P_{y/x}(t_n) \right) + A_{y/x}^{\text{CD}}(t_n)P_{xy/yx}(t_n) - \chi_{x/y}(t_n) \right],
\]

\[
P_{x/y}(t_n) = \frac{1}{N_{\text{NLE}}} \sum_{k \in K} |A_{x/y}(t_k)|^2, \quad P_{xy}(t_n) = \frac{1}{N_{\text{NLE}}} \sum_{k \in K} A_x(t_k) A_y^{*}(t_k), \quad P_{yx}(t_n) = P_{xy}^{*}(t_n),
\]

- **where the summation (weighting) interval, \( K \), is of adjustable width, \( N_{\text{NLE}} \),**

\[
K = \left\{ k : n - \left\lfloor \frac{N_{\text{NLE}}}{2} \right\rfloor + 1 \leq k \leq n + \left\lceil \frac{N_{\text{NLE}}}{2} \right\rceil \right\}
\]

Frequency-domain iXPM filter - simVSNE[0]:

\[
\tilde{A}_{iXPM}^{x/y}(\omega_n) = \kappa \left[ \tilde{A}_{CD}^{x/y}(\omega_n) \left( 2 \sum_{k=1}^{N} |\tilde{A}_{x/y}(\omega_k)|^2 + \sum_{k=1}^{N} |\tilde{A}_{y/x}(\omega_k)|^2 - \tilde{\chi}(\omega_n) \right) + \tilde{A}_{CD}^{y/x}(\omega_n) \sum_{k=1}^{N} \tilde{A}_{x/y}(\omega_k) \tilde{A}_{y/x}^*(\omega_k) \right],
\]

Applying an inverse Fourier transform, we obtain a time-domain equivalent:

\[
A_{iXPM}^{x/y}(t_n) = \kappa \left[ A_{CD}^{x/y}(t_n) \left( 2P_{x/y}(t_n) + P_{y/x}(t_n) \right) + A_{CD}^{y/x}(t_n)P_{xy/yx}(t_n) - \chi_{x/y}(t_n) \right],
\]

\[
P_{x/y}(t_n) = \frac{1}{N_{NLE}} \sum_{k\in K} |A_{x/y}(t_k)|^2, \quad P_{xy}(t_n) = \frac{1}{N_{NLE}} \sum_{k\in K} A_x(t_k)A_y^*(t_k), \quad P_{yx}(t_n) = P_{xy}^*(t_n),
\]

where the summation (weighting) interval, \(K\), is of adjustable width, \(N_{NLE}\).

An identical procedure can be done for the remaining simVSNE\(\{K\}\) filters, yielding the weighted VSNE (W-VSNE) filter array.
Time Domain VSNE

5.2) Experimental Results
Laboratorial setup of a ultra-long-haul 400G PM-16QAM transmission system implemented in collaborations CPqD, Campinas, Brazil:

- 2×32 Gbaud PM-16QAM signal;
- Propagation over a recirculating loop composed of 5×ULA (112 μm²) spans with 50 km each;
- 5 WDM superchannels spaced by 75 GHz;
- Digital Nyquist pulse shaping.

Estimated maximum reach as a function of the DBP step-size:

- The standard SSFM is not adequate for low-complexity DBP.

Weighted VSNE - Experimental Results

- Estimated maximum reach as a function of the DBP step-size:

![Graph showing maximum reach vs. step-size for SSFM and W-SSFM with CDE reach of 4700 km.]

- The standard SSFM is **not adequate** for low-complexity DBP.
- The W-SSFM shows a significantly **increased tolerance to large step-sizes**;
Weighted VSNE - Experimental Results

- Estimated maximum reach as a function of the DBP step-size:

The standard SSFM is not adequate for low-complexity DBP.

The W-SSFM shows a significantly increased tolerance to large step-sizes;
Weighted VSNE - Experimental Results

- Estimated maximum reach as a function of the DBP step-size:

![Graph showing reach vs. step-size with different lines for SSFM, W-SSFM, W-VSNE[0], and W-VSNE[1].]

- The standard SSFM is **not adequate** for low-complexity DBP.
- The W-SSFM shows a significantly increased tolerance to large step-sizes;
- The W-VSNE[1] outperforms the W-SSFM in the medium performance region;

Weighted VSNE - Experimental Results

- Estimated maximum reach as a function of the DBP step-size:

  ![Graph](image)

  - The standard SSFM is **not adequate** for low-complexity DBP.
  - The W-SSFM shows a significantly increased tolerance to large step-sizes;
  - The W-VSNE[1] outperforms the W-SSFM in the medium performance region;
  - Even with a step-size of 1000 km (**6 steps in total**) the W-VSNE[1] enables to extended the signal reach by **550 km**.

Multi-Carrier Digital Backpropagation

6.1) Numerical Implementation
Numerical implementation of the CE-DBP approach using the asymmetric SSFM:

- **SPM** is compensated by the standard SSFM over each subcarrier;
Numerical implementation of the CE-DBP approach using the asymmetric SSFM:

- SPM is compensated by the standard SSFM over each subcarrier;
- XPM is compensated by introducing coupling terms in the nonlinear step;
  * A new operator, \( \hat{W}^{(m)}(\omega) \), is responsible for the walk-off effect between subcarriers.
Numerical implementation of the CE-DBP approach using the asymmetric SSFM:

- SPM is compensated by the standard SSFM over each subcarrier;
- XPM is compensated by introducing coupling terms in the nonlinear step;
  * A new operator, \( \hat{\mathcal{W}}^{(m)}(\omega) \), is responsible for the walk-off effect between subcarriers.

Complexity is similar to that of intra-channel compensation of each subcarrier.
Multi-Carrier Digital Backpropagation

6.2) Experimental Results
Multi-Carrier DBP - Laboratorial Setup

- Laboratorial setup for **metro** and **ultra-long-haul** 400G PM-16QAM transmission system implemented in collaborations CPqD, Campinas, Brazil:

- **ULH** - 2×32 Gbaud and 3×21 Gbaud PM-16QAM signal;
  * 5 WDM superchannels in a 75 GHz slot - 5.33 b/s/Hz;

- **Metro** - 3×14 Gbaud PM-64QAM signal;
  * 1 superchannel in a 50 GHz slot - 8 b/s/Hz;

---

400G Superchannel Configurations

- Superchannel configurations for PM-16QAM:

  - $2 \times 31.5$ GBaud
    - 20 GHz
    - 35 GHz
    - 20 GHz
  - 75 GHz

  - $3 \times 21$ GBaud
    - 14.5 GHz
    - 23 GHz
    - 23 GHz
    - 14.5 GHz
  - 75 GHz

- Net spectral efficiency of 5.33 b/s/Hz;
- Designed for long-haul and ultra-long-haul applications.

400G Superchannel Configurations

- **Superchannel configurations for PM-16QAM:**

  2×31.5 GBaud
  20 GHz | 35 GHz | 20 GHz
  75 GHz

  3×21 GBaud
  14.5 GHz | 23 GHz | 23 GHz | 14.5 GHz | 114.5 GHz
  75 GHz

- **Net spectral efficiency of 5.33 b/s/Hz;**
- **Designed for long-haul and ultra-long-haul applications.**

- **Superchannel configuration for PM-64QAM - 3×14 GBaud:**

  9 GHz | 16 GHz | 16 GHz | 9 GHz
  50 GHz

- **Net spectral efficiency of 8 b/s/Hz;**
- **Designed for metro and long-haul applications** with high spectral efficiency.

Ultra-long-haul 400G propagation performance:

- after CDE, triple-carrier 400G provides an increased reach of approximately 6%;
  - good agreement with previous works based on simulation and EGN model predictions;

Ultra-long-haul 400G propagation performance:

- After CDE, triple-carrier 400G provides an increased reach of approximately 6%;
  - good agreement with previous works based on simulation and EGN model predictions;

DBP Impact on ULH 400G Performance

- Ultra-long-haul 400G propagation performance:

  - after CDE, triple-carrier 400G provides an increased reach of approximately 6%;
    - good agreement with previous works based on simulation and EGN model predictions;

  - after CE-DBP, the reach improvement provided by the triple-carrier 400G is now of 11%;
    - CE-DBP performance improves with increasing number of backpropagated subcarriers.

- The overall reach enhancement over CDE is of 1250 km (26%) and 1600 km (32%) for the dual- and triple-carrier 400G superchannels, respectively.

MC-DBP: Experimental Results for PM-64QAM

- DBP optimization and performance of 400G superchannels based on PM-64QAM:

- Maximum reach after CDE - 1100 km;

MC-DBP: Experimental Results for PM-64QAM

- DBP optimization and performance of 400G superchannels based on PM-64QAM:

- maximum reach after CDE - 1100 km;
- maximum reach after DBP - 1550 km;

MC-DBP: Experimental Results for PM-64QAM

- DBP optimization and performance of 400G superchannels based on PM-64QAM:

  - maximum reach after CDE - 1100 km;
  - maximum reach after DBP - 1550 km;
  - maximum reach after CE-DBP - 1750 km;
    - 13% over DBP and 60% over CDE.
  - The required DBP spatial resolution is very similar to that found for ULH based on PM-16QAM.

Open Research Topics on Nonlinear Equalization
A practical implementation of DBP requires some adaptation capabilities:
- Uncertainties on the link parameters;
- Non-homogenous power/gain profile;
- Other temporal variations due to temperature, bending...

Fully adaptable DBP would be too complex and very difficult to converge:
- The same happens with linear equalization (static + adaptive);

The bulk estimated nonlinearities can be compensated with a static equalizer;

A low complexity adaptive equalizer (Volterra?) optimizes the initial solution.
DBP for Subcarrier Multiplexing

- What is the best performance vs complexity trade-off?
  - Low symbol-rate per subcarrier ⇒ reduced DBP complexity per subcarrier;
  - Many subcarriers ⇒ many DBP processing chains;
  - How to efficiently deal with FWM?

- Too many coupled equations may be suboptimum;
  - Too many DBP chains - lots of FFT/IFFT pairs;
  - A fully frequency domain approach can be beneficial (VSNE?).
DBP for Subcarrier Multiplexing

- What is the best performance vs complexity trade-off?
  - Low symbol-rate per subcarrier ⇒ reduced DBP complexity per subcarrier;
  - Many subcarriers ⇒ many DBP processing chains;
  - How to efficiently deal with FWM?

- Too many coupled equations may be suboptimum;
  - Too many DBP chains - lots of FFT/IFFT pairs;
  - A fully frequency domain approach can be beneficial (VSNE?).

- The most efficient solution is likely to be based on an hybrid CE/TF-DBP approach;
  - Tradeoff between complexity per DBP chain and number of DBP chains;
DBP for Subcarrier Multiplexing

- What is the **best performance vs complexity** trade-off?
  - Low symbol-rate per subcarrier ⇒ **reduced DBP complexity** per subcarrier;
  - Many subcarriers ⇒ **many DBP processing chains**;
  - How to efficiently deal with FWM?

- Too many coupled equations may be suboptimum;
  - Too many DBP chains - lots of FFT/IFFT pairs;
  - A **fully frequency domain** approach can be beneficial (VSNE?).

- The most efficient solution is likely to be based on an **hybrid CE/TF-DBP approach**;
  - Tradeoff between **complexity per DBP chain** and **number of DBP chains**;
DBP for Subcarrier Multiplexing

- What is the best performance vs complexity trade-off?
  - Low symbol-rate per subcarrier ⇒ reduced DBP complexity per subcarrier;
  - Many subcarriers ⇒ many DBP processing chains;
  - How to efficiently deal with FWM?

- Too many coupled equations may be suboptimum;
  - Too many DBP chains - lots of FFT/IFFT pairs;
  - A fully frequency domain approach can be beneficial (VSNE?).

- The most efficient solution is likely to be based on an hybrid CE/TF-DBP approach;
  - Tradeoff between complexity per DBP chain and number of DBP chains;

- FWM will be dominant: Not all subcarriers need to be taken into account.
The FLEX-ON Project
What is Flex-ON?

- **Flexible Optical Networks** – Time Domain Hybrid QAM: DSP and Physical Layer Modelling;

- **H2020 project**: Marie Skłodowska-Curie Individual Fellowship;

  "The Marie Skłodowska-Curie actions, named after the double Nobel Prize winning Polish-French scientist famed for her work on radioactivity, support researchers at all stages of their careers, irrespective of nationality. Researchers working across all disciplines, from life-saving healthcare to ‘blue-sky’ science, are eligible for funding. The MSCA also support industrial doctorates, combining academic research study with work in companies, and other innovative training that enhances employability and career development’’.

- **24 months**: October 2015 - October 2017;

- Includes a **secondment** (3 months) with CISCO Photonics.

- Divided into 5 WPs:
  - **WP1** – Digital modulation techniques;
  - **WP2** – Simulation tools and DSP subsystems;
  - **WP3** – NL prediction and equalization tools;
  - **WP4** – Laboratorial implementation, test and validation;
  - **WP5** – Management and dissemination;
What are the Main Objectives?

- **Scientific Objectives:**
  - Optimization of digital modulation techniques for spectrally efficient transmission with high bit-rate granularity, low energy consumption and robust signal propagation;
  - Development of efficient DSP subsystems for flexible optical transceivers;
  - Development of NL prediction and equalization tools;
  - Implementation of a laboratorial testbed for validation purposes;
  - Provide new skills and career opportunities for the ER and potentiate new collaboration opportunities for the host organisation.
What are the Main Objectives?

**Scientific Objectives:**

- Optimization of *digital modulation techniques* for spectrally efficient transmission with *high bit-rate granularity*, low energy consumption and robust signal propagation;
- Development of *efficient DSP subsystems* for flexible optical transceivers;
- Development of *NL prediction and equalization tools*;
- Implementation of a *laboratorial testbed* for validation purposes;
- Provide *new skills and career opportunities* for the ER and potentiate *new collaboration opportunities for the host organisation*.

**Dissemination and Public Engagement:**

- Participate in the *European Researchers’ Night*;
- Organize an *MSCA seminar* to promote the Marie Skłodowska-Curie Actions among undergraduate students and early stage researchers;
- Design a *website* with both technical and simplified materials;
- Dissemination of optics and photonics at *high-schools*;
- Organize an *yearly workshop* on flexible optical transceivers in POLITICO;
- Organize a *workshop at an international conference*. 
Thanks for your attention!

fernando.guiomar@polito.it