Suppression of Spurious Tones in Fiber System Simulations based on the Split-Step Method

G. Bosco, A. Carena, V. Curri, R. Gaudino, P. Poggiolini

Dipartimento di Elettronica, Politecnico di Torino, Torino, ITALY

E-mail: curri@polito.it

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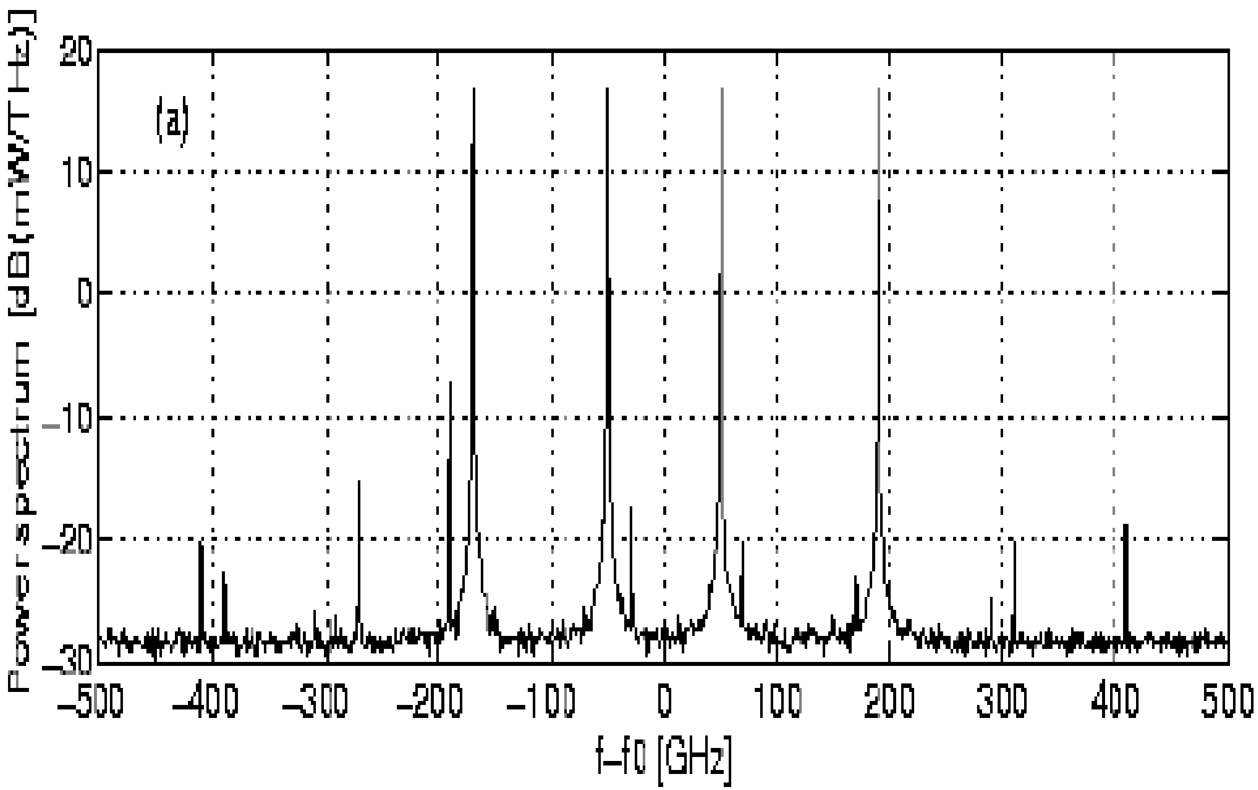
Purpose of the work

To reduce the simulation artifacts induced by the Split-Step method in optical systems simulations.



Nonlinear Schrödinger equation

The nonlinear Schrödinger equation (NLSE) governing the optical signal propagation in fiber optics can be schematically expressed as:

$$\frac{\partial \mathcal{E}(z,T)}{\partial z} = (\mathcal{L}(T) + \mathcal{N}(z))\mathcal{E}(z,T) \tag{1}$$

where:

$$\mathcal{L}(T) = \jmath \frac{\beta_2}{2}\frac{\partial^2}{\partial T^2} + \frac{\beta_3}{6}\frac{\partial^3}{\partial T^3} \tag{2}$$

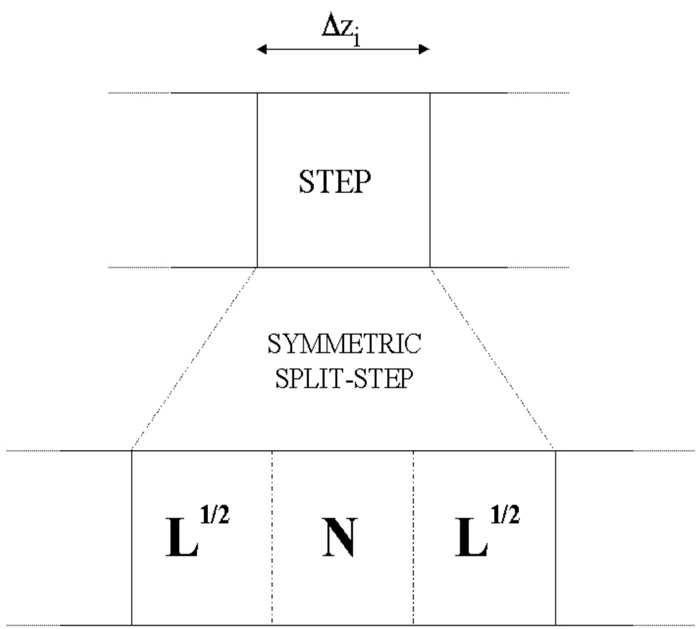
is a differential linear operator that accounts for the chromatic dispersion and

$$\mathcal{N}(z) = -\alpha + \jmath\gamma|\mathcal{E}(z)|^2 \tag{3}$$

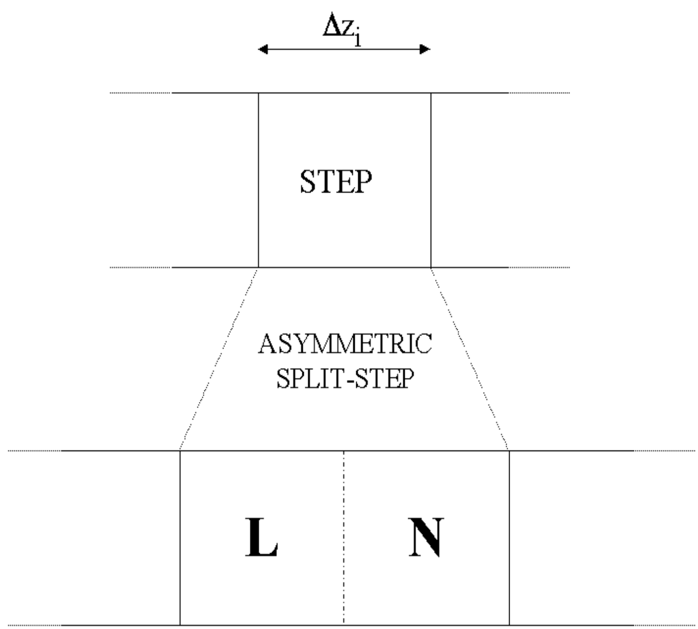
is the non-linear operator including the effects of Kerr non-linearity and fiber loss.

The split-step method (SSM)

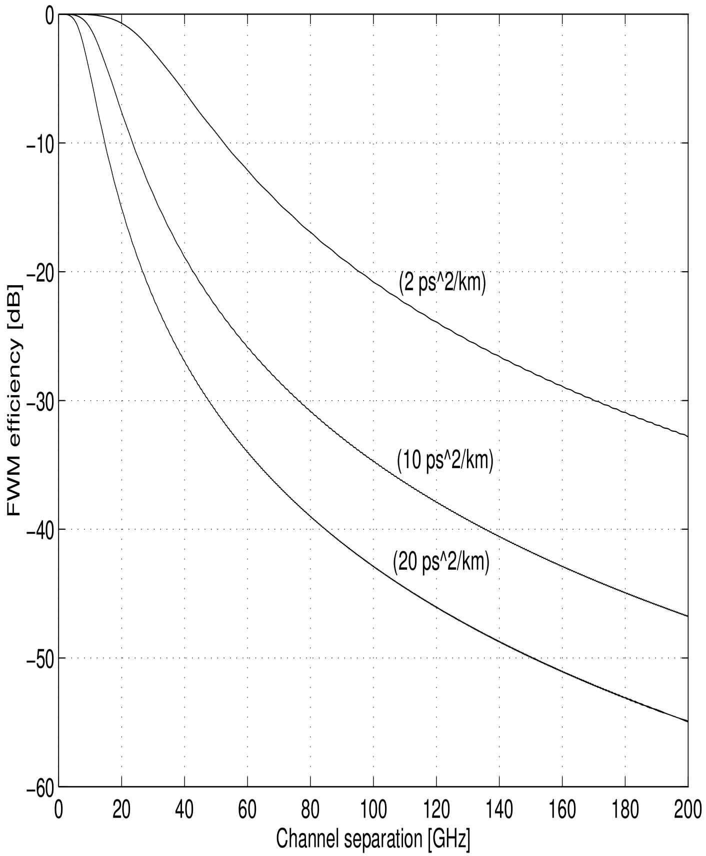
Symmetric split-step method



Asymmetric split-step method



Four-wave mixing efficiency



$$\eta = \frac{4\alpha^2}{4\alpha^2 + \Delta\beta^2}\left(1 + \frac{4\mathrm{e}^{-2\alpha L}\sin^2(\Delta\beta L/2)}{(1-\mathrm{e}^{-2\alpha L})^2}\right)$$

where:

- $\Delta\beta = 4\pi^2\beta_2\Delta f^2$ is the *phase mismatch*;
- Δf is the channel separation;
- α is the fiber loss;
- L is the span length.

Split-step and four-wave mixing

Regarding FWM, the application of the SSM is equivalent to solve the NLSE considering $\Delta\beta z'$ as a constant over each step:

$$\Delta\beta z' = \sum_{l=0}^{K-1} u(z' - S_l)\Delta z_{l+1}\Delta\beta$$

where $S_l = \sum_{p=1}^{l} \Delta z_p$.

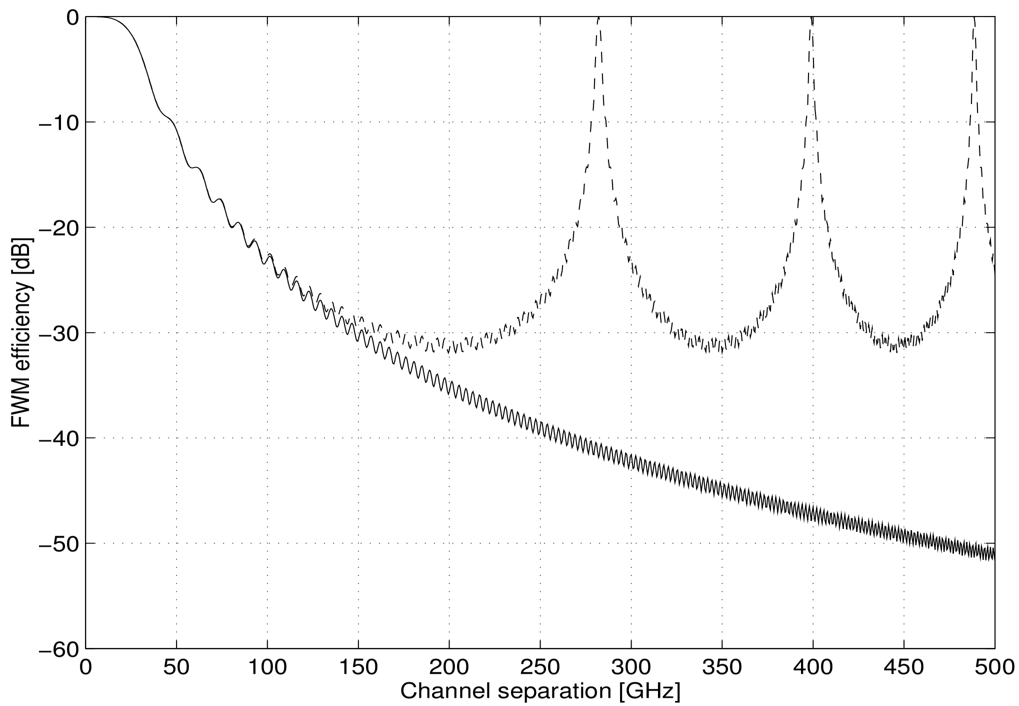
The resulting FWM efficiency altered by the SSM in a fiber span of length L, subdivided into K generic sections $\Delta z_1, \ldots, \Delta z_K$, is:

$$\eta' = \frac{\left|\sum_{l=0}^{K-1} \mathrm{e}^{-\jmath\Delta\beta S_{l+1}} \mathrm{e}^{-2\alpha S_l} \left(1 - \mathrm{e}^{-2\alpha\Delta z_{l+1}}\right)\right|^2}{\left(1 - \mathrm{e}^{-2\alpha\Delta z_{l+1}}\right)^2} .$$

The uniform step-size (USSD)

Imposing $\Delta z_l = \Delta z \; \forall \; l = 1, \ldots, K$, yields:

$$\eta'_{USSD} = \frac{1 + \mathrm{e}^{-4\alpha\ \Delta z} - 2\mathrm{e}^{-2\alpha\ \Delta z}}{1 + \mathrm{e}^{-4\alpha\ \Delta z} - 2\mathrm{e}^{-2\alpha\ \Delta z} \cos \Delta\beta\Delta z} \cdot \frac{1 + \mathrm{e}^{-4\alpha L} - 2\mathrm{e}^{-2\alpha L} \cos \Delta\beta L}{1 + \mathrm{e}^{-4\alpha L} - 2\mathrm{e}^{-2\alpha L}}$$



A novel distribution

The spurious FWM efficiency can be written as:

$$\eta' = \left| \sum_{l=1}^{K} M_l e^{\jmath \phi_l} \right|^2$$

where:

$$M_l = \frac{\left(1 - e^{-2\alpha\, \Delta z_l}\right)}{\left(1 - e^{-2\alpha L}\right)} e^{\left\{-2\alpha \sum_{p=1}^{l-1} \Delta z_p\right\}}\,, \qquad \phi_l = -\Delta\beta \sum_{p=1}^{l} \Delta z_p\,.$$

that is it can be seen as the squared magnitude of the sum of complex numbers whose absolute values are M_l and phases are ϕ_l.

Basic idea: to research a distribution of Δz_l such that $M_l = M\ \forall l$ and that phases ϕ_l are the most randomly distributed over the link, in order to induce a sort of *destructive interference*.

The logarithmic step-size (LSSD)

Imposing that $M_l = M$, $\forall l = 1, \ldots, K$, we find:

$$\Delta z_l = -\frac{1}{2\alpha} \ln \left[\frac{1 - l\delta}{1 - (l-1)\delta} \right] \quad l = 1, \ldots, K$$

where:

$$\delta = \frac{1 - \mathrm{e}^{-2\alpha L}}{K}$$

and:

$$M_l = \frac{1}{K} \;\; \forall l \; = 1, \ldots K \qquad \phi_l = \jmath \frac{\Delta\beta}{2\alpha} \ln(1 - l\delta) \quad l = 1, \ldots, K$$

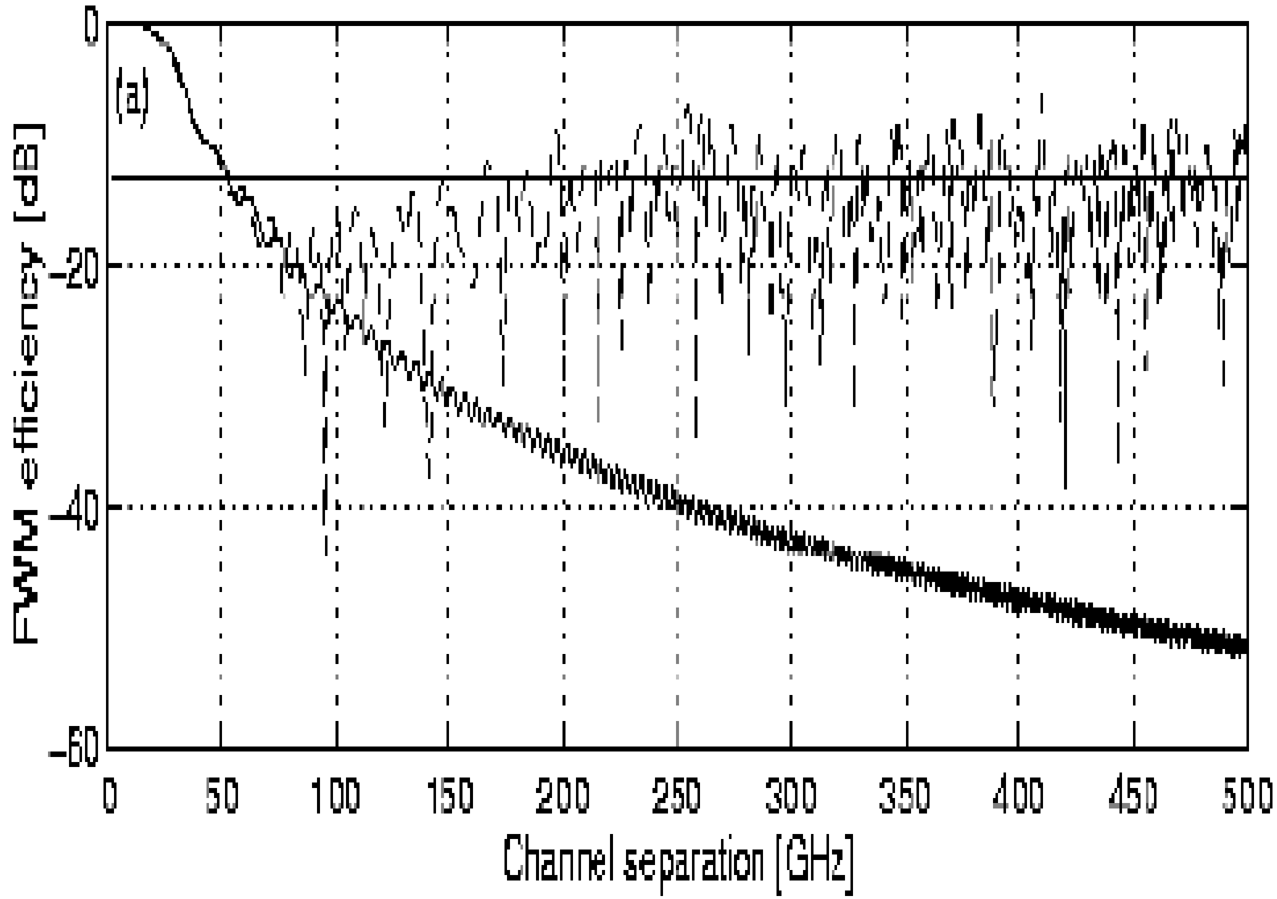
A useful result

We obtain the maximum suppression of the spurious peaks when the phases ϕ_l are totally random. In order to describe this particular case, we assume that the ϕ_l are statistically independent Gaussian random variables, and evaluate the mean value of η'_{LSSD}:

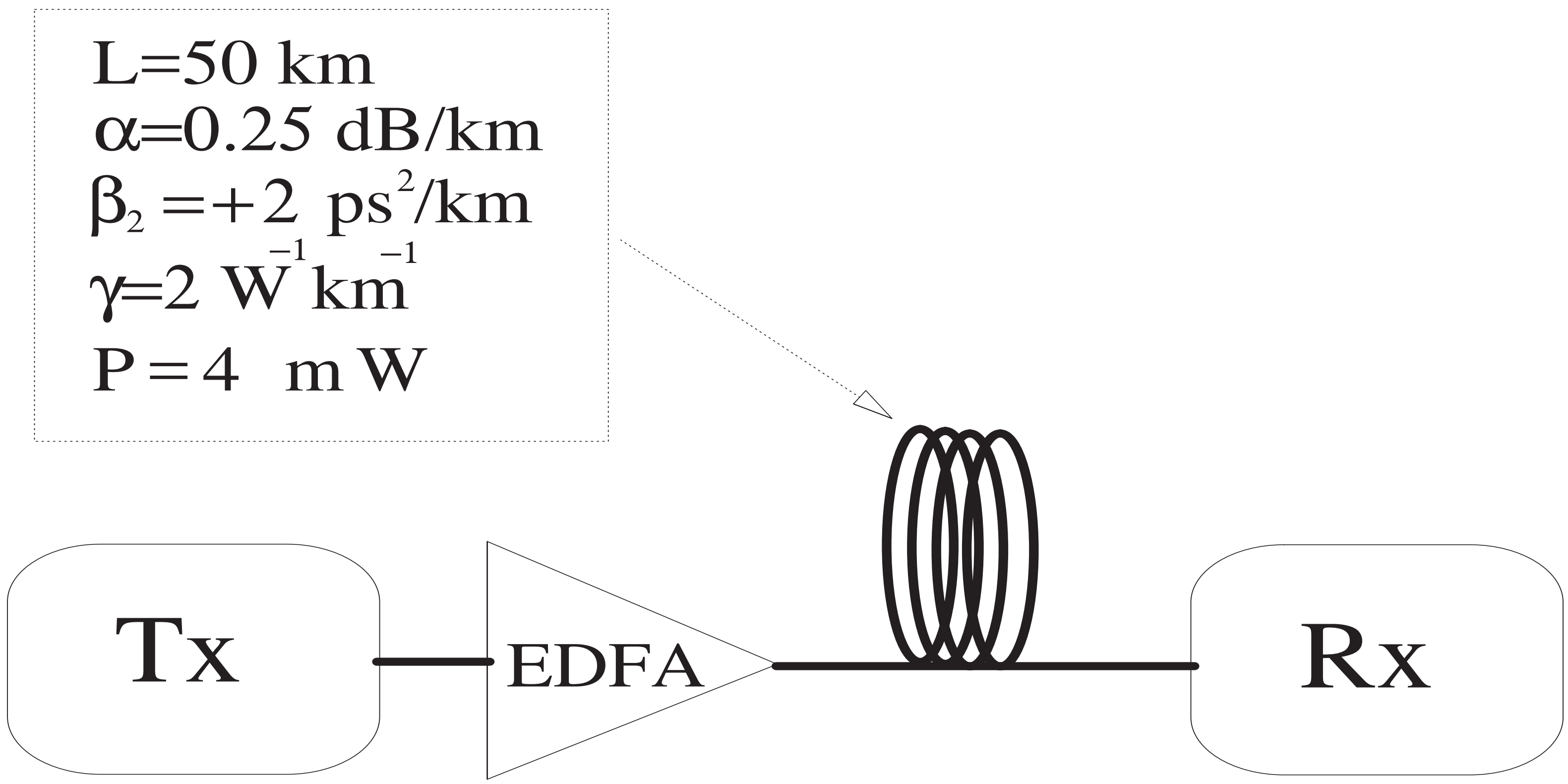
$$E\{\eta'_{LSSD}\} = \frac{1}{K}$$

where K is the number of steps. Thus, when the simulation step tends to 0, K tends to $+\infty$ and the asymptotic value of the FWM efficiency tends to the true value ($\eta'_{LSSD} = 0$).

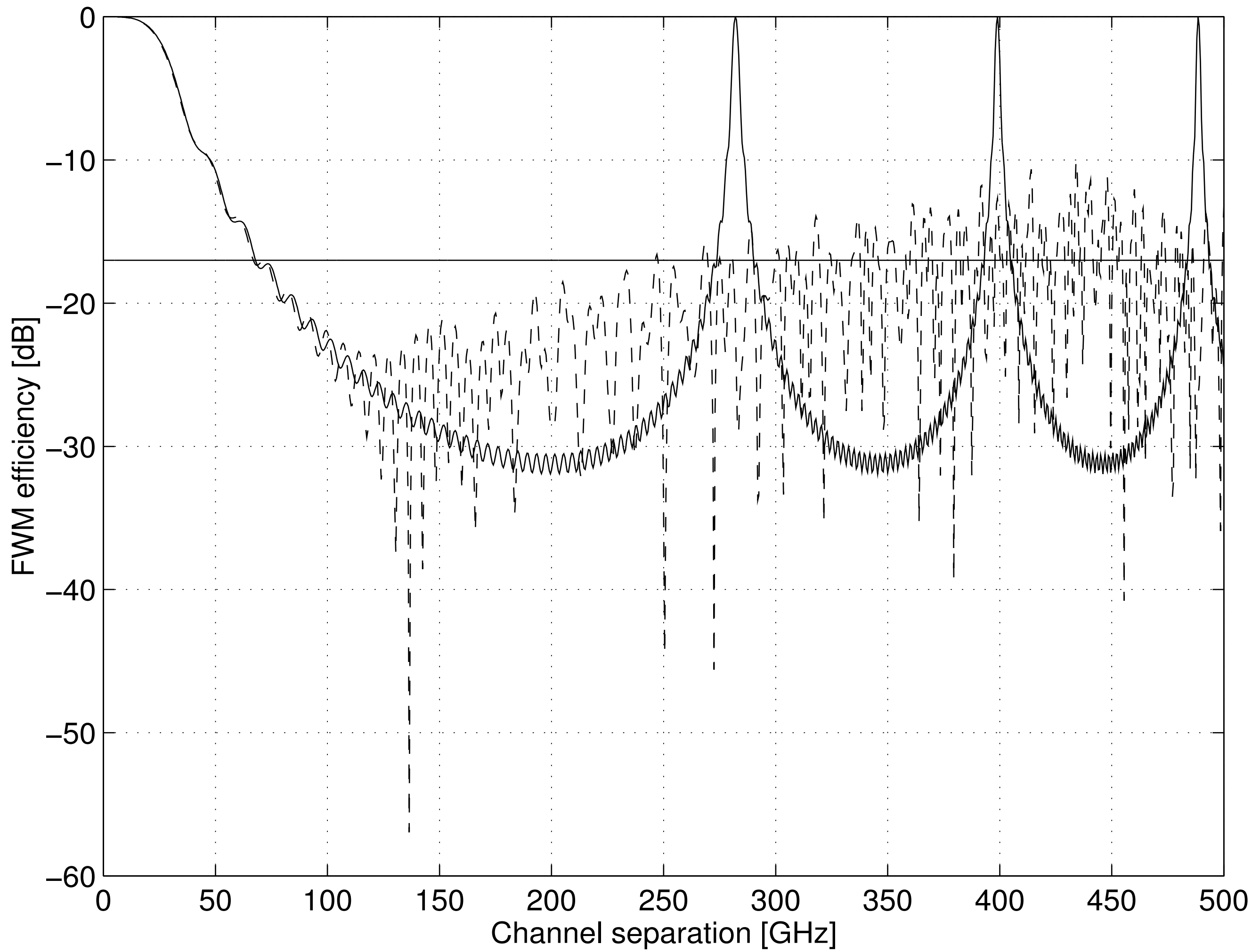
The logarithmic step-size



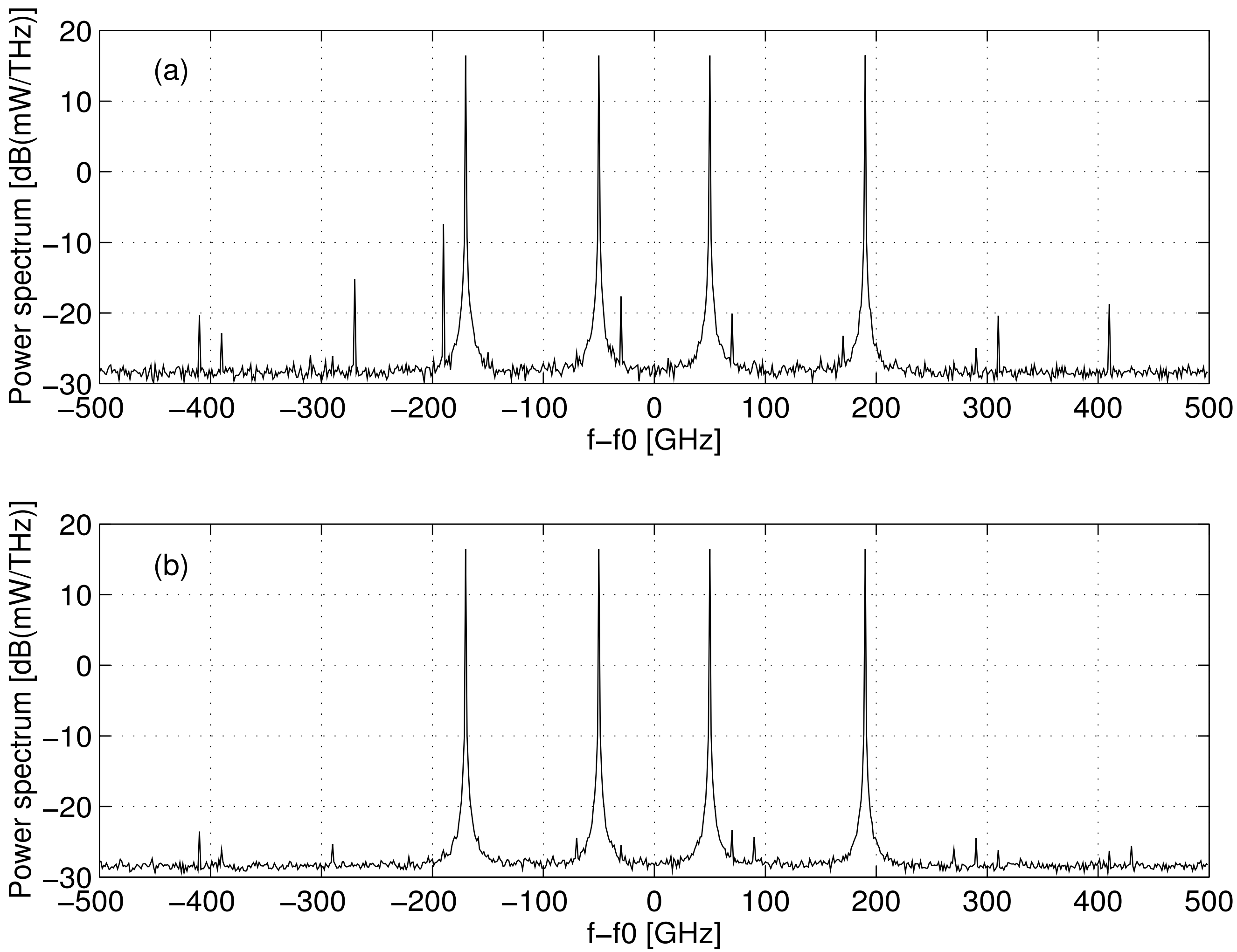
A single-span link



Uniform vs. logarithmic



Uniform vs. logarithmic



Step-size choice

A WDM signal with channels carrying the same power P has been considered. η'_{LSSD} is supposed to be equal to the floor value $1/K$, independently of the frequency, to evaluate a worst case in terms of spurious FWM level. The resulting total power of spurious FWM is:

$$P_{FWM} = \frac{\frac{3}{4} N_c^2 \gamma^2 L_e^2 \mathrm{e}^{-2\alpha L}\ P^3}{K}$$

where M is the number of generated FWM terms at the considered frequency.

It is possible to set the number of steps K that makes the power P_{FWM} of the spurious terms to be x dB below the carriers level, by imposing:

$$\frac{P_{FWM}}{P\mathrm{e}^{-2\alpha L}} < 10^{(-x/10)} \qquad \text{corresponding to} \qquad K > \frac{3}{4} N_c^2 \gamma^2 L_e^2 P^2 10^{(x/10)}\ .$$

Results

We applied the SSM, with LSSD and 20 dB of FWM suppression level, to the preceeding scenario. Only 20 steps have been required, against the 625 required to make all the spurious tones to fall outside B_W, and the 125 steps required by the "10% accuracy" criterion introduced by C. Francia in "Constant Step-Size Analysis in Numerical Simulation for Correct Four-Wave-Mixing Power Evaluation in Optical Fiber Transmission Systems", *IEEE Photonics Technology Letters*, vol.11, no.1, Jan. 1999, pp. 69-71.